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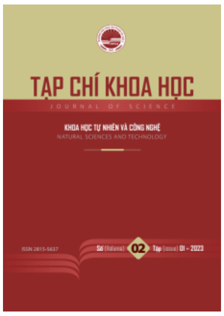


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# Analytic formulas for $(g-2)_{e,\mu}$ anomalies and decays $e_b \to e_a \gamma$ in a 3-3-1 model with inverse seesaw neutrinos

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**Abstract**

Analytic formulas for one-loop contributions to $(g-2)_{e,\mu}$ anomalies and decays amplitudes $e_b \to e_a \gamma$ are explicitly derived for the 3-3-1 model with inverse seesaw neutrinos (3-3-1ISS). Using these formulas to investigate $\Delta a_{e,\mu}$ defined as the discrepancies of $(g-2)_{e,\mu}$ anomalies between the standard and the 3-3-1ISS models, our numerical investigations showed that $\Delta a_{e,\mu}$ can reach the maximal values of around $2.5\times10^{-14}\left(1.13\times10^{-9}\right)$ and $1.4\times10^{-14}\left(0.58\times10^{-9}\right)$ for normal and inverted order schemes of neutrino oscillation data, respectively. The analytic formulas derived in this work are very useful for estimating dominant contributions to $\Delta a_{e,\mu}$ in new extended versions of the 3-3-1-1ISS model explain successfully the experimental data of $(g-2)_{e,\mu}$.

***Keywords:*** inverse seesaw mechanism, lepton flavor violating decays, beyond the standard model, etc...

## 1. Introduction

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Lepton sector is one of the most interesting subjects for experiments to search for new physics beyond the predictions of the standard model (SM). For example, the evidences of neutrino oscillation confirm that the SM must be extended. Recently, experimental data of anomalous magnetic moments of charged leptons $(g-2)_{e_a}/2 \equiv a_{e_a}$ have been updated, where the deviation between SM prediction and experiments for muon is [1]

$$\Delta a_{\mu}^{NP} \equiv a_{\mu}^{\exp} - a_{\mu}^{SM} = (251 \pm 59) \times 10^{-11}, \tag{1}$$

corresponding to $4.2\sigma$ deviation from the SM prediction [2], where $1\sigma$ corresponds to the 68% confidence interval in the normal distribution. For the electron anomaly, the deviation between the SM and experiment is $1.6\sigma$ discrepancy, namely [3]

$$\Delta a_{e}^{NP} \equiv a_{e}^{\exp} - a_{e}^{SM} = (4.8 \pm 3.0) \times 10^{-13}. \tag{2}$$

On the other hand, $\Delta a_{e,\mu}$ are strongly constrained by the experimental data of obtained from searching for the charged lepton flavor violating (cLFV) decays $e_b \to e_a\gamma$ which are obtained as [4, 5]

$$Br(\tau \to \mu\gamma) < 4.4 \times 10^{-8}, Br(\tau \to e\gamma) < 3.3 \times 10^{-8}, Br(\mu \to e\gamma) < 4.2 \times 10^{-13}. \tag{3}$$

It is well-known that constructing models beyond the standard model that explain successfully the $(g-2)$ data, the allowed regions of the parameter space must be checked to satisfy the constraints in Eq. (3). Recently work an extension of the 3-3-1 with right handed neutrinos (RHN), named as the 3-3-1 model with inverse seesaw neutrinos (3-3-1ISS), with the aim of giving an explanation the $(g-2)$ data along with explaining successfully the neutrino oscillation data through the inverse seesaw (ISS) mechanism [6]. But the numerical results that adding only three gauge singlets of neutral leptons $X_R$, the model just predicts the largest values of $\Delta a_{\mu}$ is around $108.5 \times 10^{-11}$, because of the strong cLFV constraint (3), especially $Br(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$. Adding a new charged Higgs singlet into this model is enough to show that there exist allowed regions of the parameter space satisfying all cLFV bounds (3) and explaining successfully $1\sigma$ data of $\Delta a_{\mu}$ given in Eq. (1).

In this work, we will derive analytic formulas determining the one-loop contributions to $\Delta a_{e,\mu}$ and $Br(e_b \to e_a\gamma)$. These formulas were not derived explicitly in Ref. [6]. We also use these formulas to investigate numerically $\Delta a_{e,\mu}$ in the 3-3-1ISS frame work under the cLFV constraints to look for the largest values of $\Delta a_{e,\mu}$ in both normal (NO) and inverted (IO) order schemes of neutrino oscillation data.

Our paper is organized as follows. In the section review of the 3-3-1ISS model we summary the 3-3-1ISS model discussed on Ref. [6]. We will also introduce all couplings needed to determine the one-loop amplitudes of cLFV decays $e_b \to e_a\gamma$ and the $(g-2)_{e_a}$ in the next section. The section numerical discussion we show numerical results determining all largest values of $\Delta a_{e,\mu}$. Finally, the section the conclusion contains our will remark most important results.

**2. Review of the 3-3-1 iss model**

First, the particle content of the 3-3-1ISS model [7, 8] discussed in this work is summarized as follows. The electric charge operator corresponding to the gauge group $SU(3)_L \otimes U(1)_X$ is

$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X$, where $T_{3,8}$ are diagonal $SU(3)_L$ generators and $X$ is the generator of the group $U(1)_X$. Each lepton family consists of a $SU(3)_L$ triplet $L_a = (\nu_a, e_a, N_a)_L^T \sim (3, -1/3)$ and a right-handed charged lepton $e_{aR} \sim (1, -1)$ with a=1, 2, 3. Each left-handed neutrino $N_{aL} = (N_{aR})^c$ implies a new RHN beyond the SM. Difference from the usual 3-3-1RHN model [9], the 3-3-1ISS model contains three additional right-handed neutrinos RHNs $X_{aR} = (1,0)$, a=1, 2, 3. The three Higgs triplets are introduced as

$$\rho = (\rho_1^+, \rho^0, \rho_2^+)^T \sim \left(3, \frac{2}{3}\right), \eta = (\eta_1^0, \eta^-, \eta_2^0)^T \sim \left(3, -\frac{1}{3}\right), \text{and } \chi = (\chi_1^0, \chi^-, \chi_2^0)^T \sim \left(3, -\frac{1}{3}\right)$$

with the following vacuum expectation values (VEV) of the neutral Higgs components

$$\langle\rho\rangle = \left(0, \frac{v_1}{\sqrt{2}}, 0\right)^T, \langle\eta\rangle = \left(\frac{v_2}{\sqrt{2}}, 0, 0\right)^T, \langle\chi\rangle = \left(0, 0, \frac{\omega}{\sqrt{2}}\right)^T \quad (4)$$

in order to generate all tree-level quark and lepton masses. All masses of gauge bosons are determined from the covariant kinetic term of the Higgs bosons,

$$\mathcal{L}^H = \sum_{H=\chi,\eta,\rho} (D_\mu H)^\dagger (D^\mu H),$$

with $D_\mu = \partial_\mu - igW_\mu^a T^a - ig_X T^9 X X_\mu, a = 1,2,...,8$, $T^a$ are $SU(3)_L$ generators, and the $U(1)_X$ generator $T^9 \equiv I_3/\sqrt{6}$ and $1/\sqrt{6}$ for (anti)triplets and singlets. The relations between the two gauge couplings $g$ and $g_X$ with the electric charge $e$ and sine of the Weinberg angle ($s_W^2 \simeq 0.231$) are $e = g/s_W$ and $g_X/g = 3\sqrt{2}s_W/\sqrt{3-4s_W^2}$.

The 3-3-1ISS model consists of two pairs of singly charged gauge bosons, which one of them has mass at $SU(3)_L$ scale, namely $Y_\mu^\pm = (W_\mu^6 \pm iW_\mu^7)/\sqrt{2}$ with mass $m_Y^2 = g^2(\omega^2 + v_1^2)/4$ and the ones denoted as $W^\pm$ are identified with the SM ones, namely $W^\pm = (W_\mu^1 \mp W_\mu^2)/\sqrt{2}$, leading to the following relation

$$v_1^2 + v_2^2 \equiv v^2 = (246 GeV)^2. \quad (5)$$

The two VEVs $v_1$ and $v_2$ are expressed in terms of $v$ and $\tan\beta$ defined as follows

$$t_\beta \equiv \tan\beta = \frac{v_2}{v_1}, v_1 = vc_\beta, v_2 = vs_\beta. \quad (6)$$

The Yukawa Lagrangian for generating lepton masses used in Ref. [6] is:

$$\mathcal{L}_l^Y = -h_{ab}^e \bar{L}_a \rho e_{bR} + h_{ab}^\nu \varepsilon^{ijk} (\bar{L}_a)_i (L_b)_j^c \rho_k^* - y_{ba}^\chi \bar{\chi}_{bR} \chi^\dagger L_a - \frac{1}{2}(\mu_\chi)_{ab} \bar{\chi}_{aR} (X_{bR})^c + H.\text{c.}, \quad (7)$$

where $a,b = 1,2,3$ are the number of $\chi_R$. The first term generates charged lepton masses, $m_{e_a} \equiv \frac{h_{ab}^e v_1}{\sqrt{2}} \delta_{ab}$.

In the basis $\nu_L' = (\nu_L, N_L, (X_R)^c)^T$ and $(\nu_L')^c = ((\nu_L)^c, (N_L)^c, X_R)^T$, the Lagrangian (7) leads to neutrino mass matrix $M^\upsilon$ with order $9\times 9$ written in a block form as follows

$$-L^{\nu}_{\text{mass}} = \frac{1}{2}\left(\nu'_L\right)^c M^{\nu}\nu'_L + H.c., \tag{8}$$

where $M^{\nu} = \begin{pmatrix} \mathcal{O} & m_D^T & \mathcal{O} \\ m_D & \mathcal{O} & M_R^T \\ \mathcal{O} & M_R^T & \mu_X^{\dagger} \end{pmatrix}$,

with $\left(M_R\right)_{ab} \equiv Y^{\chi}_{ab}\omega/\sqrt{2}, \left(m_D\right)_{ab} \equiv \sqrt{2}h^{\nu}_{ab}v_1 = -\left(m_D\right)_{ab}$ being antisymmetric with $a,b = 1,2,3$. The matrix $\mu_X$ defined in Eq. (7) is symmetric, hence can be diagonalized by a unitary transformation $U_X$, namely $U_X^T\mu_X U_X = \text{diag}\left(\mu_{X_1},\mu_{X_2},\mu_{X_3}\right)$. The matrix $U_X$ will be absorbed in the redefinitions the neutral fermion states $X_{aR}$ through the following transformations: $\mu_X \to U^T\mu_X U_X, X_{\text{aR}} \to U_X^T X_{\text{aR}}$, and $y^{\chi} \to U_X^T y^{\chi}$.

The mass matrix $M^{\nu}$ is diagonalized by a 9×9 unitary matrix $U^{\nu}$,

$$U^{\nu T}M^{\nu}U^{\nu} = \hat{M}^{\nu} = \text{diag}\left(m_{n_1}, m_{n_2}, ..., m_{n_9}\right) = \text{diag}\left(\hat{m}_{\nu}, \hat{M}_N\right), \tag{9}$$

which are masses of the nine physical neutrino states $n_{iL}$. Namely, the masses of the three active neutrinos $n_{aL}$ and six extra neutrinos $n_{IL}$ are denoted as $\hat{m}_{\nu} = diag\left(m_{n_1}, m_{n_2}, m_{n_3}\right)$ and $\hat{M}_N = diag\left(m_{n_4}, m_{n_5}, ..., m_{n_9}\right)$, respectively. The relations between the flavor and physical base are

$$\begin{cases} n'_L = U^{\nu}n_L \\ \left(n'_L\right)^c = U^{\nu *}\left(n_L\right)^c \equiv U^{\nu *}n_R, \end{cases} \tag{10}$$

where $n_L \equiv \left(n_{1L}, n_{2L}, ..., n_{9L}\right)^T$. The majorana states are $n_i \equiv \left(n_{iL}, \left(n_{iL}\right)^c\right)^T$.

The ISS block form of the mass matrix corresponding to the general seesaw form through the following transformations

$$M_D = \begin{pmatrix} m_D \\ \mathcal{O}_3 \end{pmatrix}, \text{M}_N = \begin{pmatrix} \mathcal{O}_3 & M_R^T \\ M_R & \mu_X \end{pmatrix}. \tag{11}$$

Then using the well-known formula of $U^{\nu}$ to construct all ISS relations,

$$U^{\nu} = \Omega\begin{pmatrix} U_{PMNS} & \mathcal{O}_{3\times 6} \\ \mathcal{O}_{6\times 3} & V \end{pmatrix}, \Omega \simeq \begin{pmatrix} I_3 - \frac{1}{2}RR^{\dagger} & R \\ -R^{\dagger} & I_6 - \frac{1}{2}R^{\dagger}R \end{pmatrix}, \tag{12}$$

where R, V are 3 x 6, 3 x 6 matrices, respectively. In particularly, inserting Eq. (12) into the Eq. (9) with $\Omega^T \text{M}^{\nu}\,\Omega = \text{diag}\left(U^*_{PMNS}\hat{m}_{\nu}U^{\dagger}_{PMNS}, V^*\hat{M}_N V^{\dagger}\right)$ will lead to three following independent equations

$$\left(11\right) \to U^*_{PMNS}\hat{m}_{\nu}U^{\dagger}_{PMNS} = -R^*M_D\left(1 - \frac{1}{2}RR^{\dagger}\right) + \left[\left(1 - \frac{1}{2}R^*R^T\right)M_D^T - R^*M_N\right]\left(-R^{\dagger}\right),$$

$$\left(22\right) \to V^*\hat{M}_N V^{\dagger} = \left(1 - \frac{1}{2}R^TR^*\right)M_D R + \left[R^TM_D^T + \left(1 - \frac{1}{2}R^TR^*\right)M_N\right]\left(1 - \frac{1}{2}R^{\dagger}R\right),$$

$$(12) \to \mathcal{O}_{3\times 6} = -R^{*}M_{D}R + \left[\left(1-\frac{1}{2}R^{*}R^{T}\right)M_{D}^{T} - R^{*}M_{N}\right]\left(1-\frac{1}{2}R^{\dagger}R\right). \tag{13}$$

Find the leading part of the entry (12) in Eq. (13), then inserting into two remaining entries (11) and (22), we obtain the following relations without any inverse matrix forms

$$M_{D}^{T} = \mathrm{R}^{*}\mathrm{M}_{N}, m_{\nu} \equiv U_{PMNS}^{*}\hat{m}_{\nu}\,\mathrm{U}_{PMNS}^{\dagger} = \mathrm{R}^{*}\mathrm{M}_{N}\mathrm{R}^{\dagger}, \tag{14}$$

$$V^{*}\hat{M}_{N}V^{\dagger} = M_{N} + \frac{1}{2}\mathrm{M}_{N}\mathrm{R}^{\dagger}R + \frac{1}{2}R^{T}\,\mathrm{R}^{*}\mathrm{M}_{N}. \tag{15}$$

Denote that $R = (R_1 \quad R_2)$, where $R_{1,2}$ are two submatrices of $R$ with the same order 3 x 3, the leading part of (12) gives

$$M_{D}^{T} = \mathrm{R}^{*}\mathrm{M}_{N} \Leftrightarrow \begin{cases} m_{D}^{T} = R_{2}^{*}M_{R}, \\ R_{1}^{*}M_{R}^{T} + R_{2}^{*}\mu_{X} = \mathcal{O}_{3}. \end{cases} \tag{16}$$

Inserting Eq. (16) into the first line of Eq. (13) and keeping only the dominant contribution, we have $R^{*}M_{D} = M_{D}R^{\dagger} = R^{*}M_{N}R^{\dagger} \gg R^{*}M_{D}RR^{\dagger},...,$ and $U_{PMNS}^{*}\hat{m}_{\nu}U_{PMNS}^{\dagger} = -R^{*}M_{N}R^{\dagger} = -M_{D}^{T}R^{\dagger} = R_{2}^{*}\mu_{X}R_{2}^{\dagger}$. Now it is easily to derive the last entry (12) of Eq. (13) defining the heavy neutrino masses. We summarize here the most important ISS relations

$$\begin{aligned} &R_{2}^{*} = m_{D}^{T}\,\mathrm{M}_{R}^{-1}, R_{1}^{*} = -R_{2}\mu_{X}\left(M_{R}^{-1}\right)^{T} \simeq \mathcal{O}_{3}, \\ &\mathrm{m}_{\nu} \equiv R_{2}^{*}\mu_{X}R_{2}^{\dagger} = U_{PMNS}^{*}\hat{m}_{\nu}U_{PMNS}^{\dagger} = m_{D}^{T}M_{R}^{-1}\mu_{X}\left(M_{R}^{-1}\right)^{T}m_{D} = m_{D}^{T}M^{-1}m_{D}. \end{aligned} \tag{17}$$

where $M^{-1} \equiv M_{R}^{-1}\mu_{X}\left(M_{R}^{-1}\right)^{T}$.

Based on Eq. (15), a further estimation of the heavy neutrinos mass can be calculated analytically, namely $V^{*}\hat{M}_{N}V^{\dagger} \simeq M_{N}$. Using the well-known formulas that $M_R$ is always diagonalized by two unitary transformations $V_L$ and $V_R$ [10]

$$V_{L}^{T}M_{R}V_{R} = \hat{M}_{R} = diag\left(M_{1}, \quad M_{2}, \quad M_{3}\right), \tag{18}$$

where all $\hat{M}_{1,2,3}$ are always positive. Therefore, we accept that $M_R$ is expresses in terms of $\hat{M}_{R}, V_{L,R}$ as free parameters. We can prove that $V$ in Eq. (20) can be found as follows

$$V = \frac{1}{\sqrt{2}}\begin{pmatrix} V_{R} & \mathcal{O}_{3\times 3} \\ \mathcal{O}_{3\times 3} & V_{L} \end{pmatrix}\begin{pmatrix} I_{3} & iI_{3} \\ I_{3} & -iI_{3} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} V_{R} & iV_{R} \\ V_{L} & -iV_{L} \end{pmatrix}, \tag{19}$$

leading to the equality that $V^{T}M_{N}V = diag\left(\hat{M}_{R}, \quad \hat{M}_{R}\right)$. As a consequence, for any qualitatively estimations, we will use the crude approximations that $m_{n_{a+3}} = m_{n_{a+6}} \simeq \hat{M}_{a}$ with a = 1, 2, 3; $R_{1} \simeq \mathcal{O}_{3}$, and

$$U^{\nu} \simeq \begin{pmatrix} \left(I_{3} - \frac{1}{2}R_{2}R_{2}^{\dagger}\right)U_{PMNS} & \frac{1}{\sqrt{2}}R_{2}V_{L} & -\frac{i}{\sqrt{2}}R_{2}V_{L} \\ \mathcal{O}_{3} & \frac{V_{R}}{\sqrt{2}} & \frac{iV_{R}}{\sqrt{2}} \\ -R_{2}^{\dagger}U_{PMNS} & \left(I_{3} - \frac{1}{2}R_{2}R_{2}^{\dagger}\right)\frac{V_{R}}{\sqrt{2}} & \left(I_{3} - \frac{1}{2}R_{2}R_{2}^{\dagger}\right)\frac{-iV_{R}}{\sqrt{2}} \end{pmatrix}. \tag{20}$$

Using numerical values of $m_\nu$ obtained from the neutrino oscillation data, we can determine all independent parameters $x_{12}, \mathrm{x}_{13}$ and three entries of $M^{-1}$ [7, 8], namely the Dirac mass matrix has an antisymmetric form as follows

$$m_D = z \times \tilde{m}_D, \tilde{m}_D = \begin{pmatrix} 0 & x_{12} & x_{13} \\ -x_{12} & 0 & 1 \\ -x_{13} & -1 & 0 \end{pmatrix}, \tag{21}$$

where $\mathrm{z} = \sqrt{2}\mathrm{v}_1 \mathrm{h}^{\mathrm{v}}_{23}$ is assumed to be positive real, and

$$x_{13} = \frac{(m_\mathrm{v})^2_{13} - (m_\mathrm{v})_{11}(m_\mathrm{v})_{33}}{(m_\mathrm{v})_{13}(m_\mathrm{v})_{23} - (m_\mathrm{v})_{12}(m_\mathrm{v})_{33}}, x_{12} = \frac{(m_\mathrm{v})_{12}(m_\mathrm{v})_{13} - (m_\mathrm{v})_{11}(m_\mathrm{v})_{23}}{(m_\mathrm{v})_{13}(m_\mathrm{v})_{23} - (m_\mathrm{v})_{12}(m_\mathrm{v})_{33}}, \tag{22}$$

and $\mathrm{Det}[m_\nu = 0]$. In addition, the non-diagonal entries of $M^{-1}$ and $\mu_{X,a}\,(a = 1,2,3)$ are functions of the diagonal entries $M^{-1}_{11,22,33}$, which we will not show explicitly here. We emphasize that the structure of $m_D$ given in Eqs. (21) and (22) is enough to obtain the neutrino oscillation data from the ISS relations. On the other hand, $|R_2| \ll 1$ is necessary to guarantee that the ISS relations are consistent with the result derived directly using the total mass and mixing matrices.

## 3. The analytic formulas for $(g-2)_{e,\mu}$ anomalies and decays $e_b \to e_a\gamma$ in 3-3-1ISS model

The one-loop form factors relating with $W^\pm$ and $Y$ bosons are detailly calculated and obtained as

$$\begin{aligned} c_{(ab)\mathrm{R}}(\mathrm{W}) &= \frac{eg^2}{32\pi^2 m^2_\mathrm{W}}\left[-\frac{5}{12}\left[\delta_{ab} - \left(R_2 R_2^\dagger\right)_{ab}\right] + \sum_{e=1}^{3}(R_2 V_L)_{ae}(R_2 V_L)^*_{be} F_V\left(x'_{\mathrm{W},e}\right)\right], \\ c_{(ab)\mathrm{R}}(Y) &= \frac{eg^2}{32\pi^2 m^2_Y}\sum_{i=1}^{9}(V_R)^*_{ae}(V_R)_{be} F_V\left(x'_{Y,e}\right), \end{aligned} \tag{23}$$

where $x'_{\mathrm{v},e} \equiv m^2_{n_e+3} \setminus m^2_v,\ \ v = \mathrm{W}, \mathrm{Y},\ c_{(ba)\mathrm{R}}(v) = c_{(ab)\mathrm{R}}(v)[a \to b, b \to a] \times \dfrac{m_{e_a}}{m_{e_b}}$,

$$F_V(x) = -\frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3\ln(x)}{24(x-1)^4}, \tag{24}$$

$\mathrm{e} = \sqrt{4\pi\alpha_{em}}$ being the electromagnetic coupling constant, and $g = e/s_\mathrm{W}$. Because $\lim_{x\to 0} F_V(x) = -5/12$, and $m^2_{n_i}/m^2_\mathrm{W} \simeq 0\ (i = 1,2,3)$ hence we have $F_V(x_{\mathrm{W,i}}) = F_V(x_{Y,i}) = -5/12$ with $i = 1,2,3$. In addition, $|F_V(x)| \le 5/12$, hence $\left|c_{(ab)R}(Y)\right| \le 5/12 \times \left(eg^2/\left(32\pi^2 m^2_\mathrm{W}\right)\right) \times m^2_\mathrm{W}/m^2_Y$. This implies that the one-loop contribution from $Y$ to $(g-2)_\mu$ anomaly is $a_\mu(Y) \le (m^2_\mathrm{W}/m^2_Y) \times a^{SM}_\mu(\mathrm{W}) \le 10^{-11}$ with $m_Y > 1\,\mathrm{TeV}$. Therefore, $a_{e_a}(Y)$ will be ignored in the numerical calculations.

Regarding to the contributions from singly charged Higgs bosons, the relations between the original and mass eigenstates of the charged Higgs bosons are [11]

$$\begin{pmatrix} \eta^\pm \\ \rho^\pm \end{pmatrix} = \begin{pmatrix} -s_\beta & c_\beta \\ c_\beta & s_\beta \end{pmatrix}\begin{pmatrix} G^\pm_\mathrm{W} \\ H^\pm_1 \end{pmatrix},\ \begin{pmatrix} \rho^\pm_2 \\ \chi^\pm \end{pmatrix} = \begin{pmatrix} -s_\theta & c_\theta \\ c_\theta & s_\theta \end{pmatrix}\begin{pmatrix} G^\pm_Y \\ H^\pm_2 \end{pmatrix}, \tag{25}$$

where $t_\theta = v_1/\omega \ll 1$. Relevant Lagrangian of charged Higgs bosons is

$$\mathcal{L}^{\ell nH} = -\frac{g}{\sqrt{2}m_{\mathrm{W}}}\sum_{k=1}^{2}\sum_{a=1}^{3}\sum_{i=1}^{9} H_k^+ \overline{n}_i \left(\lambda_{ai}^{L,k} P_L + \lambda_{ai}^{\mathrm{R},k} P_R\right) \mathrm{e}_a + \mathrm{H.c}, \tag{26}$$

Here

$$\lambda_{ai}^{L,1} = \mathrm{s}_\beta \mathrm{z}_0 \sum_{c=1}^{3} \left(\tilde{m}_D\right)_{ca} U^{\nu}_{(c+3)i} \simeq \mathrm{s}_\beta \mathrm{z}_0 \sum_{c=1}^{3} \begin{bmatrix} 0 & i \le 3 \\ \dfrac{\left(\tilde{m}_D V_R\right)_{a(i-3)}}{\sqrt{2}} & 3 < i \le 6, \\ \dfrac{i\left(\tilde{m}_D V_R\right)_{a(i-6)}}{\sqrt{2}} & 6 < i \le 9 \end{bmatrix}$$

$$\lambda_{ai}^{R,1} = m_{e_a} U^{\nu *}_{ai} t_\beta \simeq m_{e_a} t_\beta \begin{bmatrix} \left[\left(I_3 - \frac{1}{2} R_2 R_2^\dagger\right)^* U^*_{PMNS}\right]_{ai} & i \le 3 \\ \left(\frac{1}{\sqrt{2}} R_2 V_L\right)^*_{a(i-3)} & 3 < i \le 6, \\ \left(\frac{-i}{\sqrt{2}} R_2 V_L\right)^*_{a(i-6)} & 6 < i \le 9 \end{bmatrix}$$

$$\lambda_{ai}^{L,2} = -c_\theta z_0 \sum_{c=1}^{3} \begin{bmatrix} \left[\tilde{m}_D \left(I_3 - \frac{1}{2} R_2 R_2^\dagger\right) U_{PMNS}\right]_{ai} & i \le 3 \\ \frac{1}{\sqrt{2}}\left(\tilde{m}_D R_2 V_L\right)_{a(i-3)} & 3 < i \le 6, \\ \frac{1}{\sqrt{2}}\left(-i\tilde{m}_D R_2 V_L\right)_{a(i-6)} & 6 < i \le 9 \end{bmatrix}$$

$$\lambda_{ai}^{R,2} = \frac{m_{e_a} c_\theta U^{\nu *}_{(a+3)i}}{c_\beta} \simeq \frac{m_{e_a} c_\theta}{c_\beta} \times \begin{bmatrix} 0 & i \le 3 \\ \dfrac{\left(V_R^*\right)_{a(i-3)}}{\sqrt{2}} & 3 < i \le 6, \\ \dfrac{-i\left(V_R^*\right)_{a(i-6)}}{\sqrt{2}} & 6 < i \le 9 \end{bmatrix} \tag{27}$$

where $\mathrm{z}_0 = \sqrt{2}\,\mathrm{v}\mathrm{h}^{\mathrm{v}}_{23} = \mathrm{z}/\mathrm{c}_\beta$, and $\left(M_R\right)_{\mathrm{ij}} = z\left(\tilde{M}_R\right)_{\mathrm{ij}}$. We have ignored the small part proportional to $\mathrm{t}_0^2 \simeq \mathrm{m}_{\mathrm{W}}^2/\mathrm{m}_{\mathrm{Y}}^2 = \mathrm{v}^2/\omega^2 \ll 1$ in the last expression of $\lambda_{ai}^{L,2}$. We also fix $c_\theta \simeq 1$ and $t_\theta = s_\theta = 0$ from now on. From Eq. (18), we see that

$$V_L^T \tilde{M}_R V_R = \tilde{M}_R z^{-1} = diag\left(k_1, k_2, k_3\right) \equiv \hat{k}, \tag{28}$$

with $k_a > 1$ for all $a = 1,2,3$. Now, it can be proved that $R_2 V_L = \tilde{m}_D^\dagger V_R^* \hat{k}^{-1}$ and $R_2 R_2^\dagger = -\tilde{m}_D^* V_R^* \hat{k}^{-2} V_R^T \tilde{m}_D$ [6], hence $V_L$ does not affect all couplings given in Eq. (27). Therefore, $V_L = I_3$ will be fixed from now on.

Similarly, the one-loop form factors relating with $H_k^\pm$ are obtained as

$$c_{(ab)\mathrm{R}}\left(H_k^{\pm}\right)=r_k\sum_{i=1}^{9}\left[\lambda_{ai}^{L,k*}\lambda_{bi}^{\mathrm{R},k}m_{n_i}F_H\left(x_{i,k}\right)+\left(m_{e_b}\lambda_{ai}^{L,k*}\lambda_{bi}^{\mathrm{L},k}+m_{e_a}\lambda_{ai}^{R,k*}\lambda_{bi}^{\mathrm{R},k}\right)\tilde{F}_H\left(x_{i,k}\right)\right],$$

$$c_{(ba)\mathrm{R}}\left(H_k^{\pm}\right)=c_{(ab)R}\left(H_k^{\pm}\right)\left[a\to b,b\to a\right]\times\frac{m_{e_a}}{m_{e_b}},\tag{29}$$

where $b\geq a,\ \ r_k\equiv eg^2/\left(32\pi^2m_{\mathrm{W}}^2m_{e_b}m_{H_k}^2\right),\ \ x_{i,k}\equiv m_{n_i}^2/m_{H_k}^2$, and

$$F_H\left(x\right)=-\frac{1-x^2+2x\ln\left(x\right)}{4\left(x-1\right)^3},\tilde{F}_H\left(x\right)=-\frac{-1+6x-3x^2-2x^3+6x^2\ln\left(x\right)}{24\left(x-1\right)^4}.\tag{30}$$

The factor $r_k\sim 1/m_{H_k}^2$, hence large $\mathrm{Re}\left[c_{\mathrm{aa}}\right]\sim a_{e_a}$ supporting experimental data prefers small $m_{H_k}$. In contrast, $Br\left(e_b\to e_a\gamma\right)$ requires both small $\left|c_{(ab)R}\right|$ and $\left|c_{(ba)R}\right|$, hence there must exits the strongly cancellation among different parts in these terms. Because $\lim_{x\to 0}\left[x^{1/2}F_H\left(x\right)\right]=0$ and $\lim_{x\to 0}\tilde{F}_H\left(x\right)=1/24$, we will fix $m_{n_i}F_H\left(x_{i,k}\right)=0$ and $\tilde{F}_H\left(x_{i,k}\right)=1/24$ for $i=1,2,3$ in case active neutrinos which owning $x_{i,k}\simeq 0$. In addition, the total mixing matrix given in Eq. (20) results in that $m_{n_a+3}=m_{n_a+6}\simeq\tilde{M}_a=z_0c_\beta k_a$. Consequently, one-loop contributions to $c_{(ab)R}$ and $c_{(ba)R}$ from singly charged Higgs bosons $H_k^{\pm}$ are derived approximately as follows

$$\sum_{i=1}^{9}\lambda_{ai}^{L,1*}\lambda_{bi}^{R,1}m_{n_i}F_H\left(x_{i,1}\right)\simeq\left(-s_\beta^2z_0^2m_{e_b}\right)\sum_{e=1}^{3}\left[\left(\tilde{m}_D^*V_R^*\right)_{ae}\left(\tilde{m}_DV_R\hat{k}^{-1}\right)_{be}k_eF_H\left(x_{e,1}^{'}\right)\right],$$

$$\sum_{i=1}^{9}\lambda_{ai}^{L,1*}\lambda_{bi}^{\mathrm{L},1}m_{n_i}\tilde{F}_H\left(x_{i,1}\right)\simeq s_\beta^2z_0^2\sum_{e=1}^{3}\left[\left(\tilde{m}_DV_R\right)_{ae}^*\left(\tilde{m}_DV_R\right)_{be}\tilde{F}_H\left(x_{e,1}^{'}\right)\right],$$

$$\sum_{i=1}^{9}\lambda_{ai}^{\mathrm{R},1*}\lambda_{bi}^{R,1}\tilde{F}_H\left(x_{i,1}\right)\simeq\left(m_{e_a}m_{e_b}t_\beta^2\right)\left\{\frac{1}{24}\left[\delta_{ab}-\left(R_2R_2^{\dagger}\right)_{ab}\right]\right.$$
$$\left.+\sum_{e=1}^{3}\left[\left(\tilde{m}_D^*V_R^*\hat{k}^{-1}\right)_{ae}\left(\tilde{m}_DV_R\hat{k}^{-1}\right)_{be}\tilde{F}_H\left(x_{e,1}^{'}\right)\right]\right\},$$

$$\sum_{i=1}^{9}\lambda_{ai}^{L,2*}\lambda_{bi}^{R,2}m_{n_i}F_H\left(x_{e,2}^{'}\right)\simeq m_{e_b}z_0^2\sum_{e=1}^{3}\left[\left(\tilde{m}_D^*\tilde{m}_DV_R\hat{k}^{-1}\right)_{ae}\left(V_R^*\right)_{be}\hat{k}_eF_H\left(x_{e,2}^{'}\right)\right],$$

$$\sum_{i=1}^{9}\lambda_{ai}^{L,2*}\lambda_{bi}^{\mathrm{L},2}m_{n_i}\tilde{F}_H\left(x_{i,2}\right)\simeq z_0^2\left\{-\frac{1}{24}\left[\left(\tilde{m}_D^*\tilde{m}_D\right)_{ab}-\left(\tilde{m}_D^*R_2^*R_2^T\tilde{m}_D\right)_{ab}\right]+\sum_{e=1}^{3}\left[\left(\tilde{m}_D\tilde{m}_D^*V_R\hat{k}^{-1}\right)_{ae}\left(\tilde{m}_D^*\tilde{m}_DV_R^*\hat{k}^{-1}\right)_{be}\tilde{F}_H\left(x_{e,2}^{'}\right)\right]\right\},$$

$$\sum_{i=1}^{9}\lambda_{ai}^{\mathrm{R},2*}\lambda_{bi}^{\mathrm{R},2}\tilde{F}_H\left(x_{i,2}\right)\simeq\frac{m_{e_a}m_{e_b}}{c_\beta^2}\sum_{e=1}^{3}\left(V_R\right)_{ae}\left(V_R\right)_{be}^*\tilde{F}_H\left(x_{e,2}^{'}\right),\tag{31}$$

where $x_{e,k}^{'}\simeq x_{e+3,k}$. We can see that the structure of $\tilde{m}_D$ affects strongly on dominant terms appearing in Eq. (31). A first crude estimation is that $Br\left(e_b\to e_a\gamma\right)\sim\left|\left(\tilde{m}_D\right)_{ab}\right|^4$ while $\left|\Delta a_{e_a}\right|\sim\mathrm{Re}\left[\left|\left(\tilde{m}_D\right)_{aa}\right|^2\right]$, hence the structure (21) will lead to a strict relation between $Br\left(e_b\to e_a\gamma\right)$ and

$\Delta a_{e_a}$, unless strongly cancellation among many parts appearing in $c_{(ab)R}$ to result in $Br\left(e_b \to e_a\gamma\right)$ satisfying all cLFV constraints. Therefore, scanning the parameter space in the numerical investigation to find out these points is challenging.

For convenience, the total one-loop contribution $c_{(ab)R}$ is separated into two parts relating with charged Higgs and gauge bosons,

$$c_{(ab)R}\left(V\right)=c_{(ab)R}\left(W\right)+c_{(ab)R}\left(Y\right),\;\; c_{(ab)R}\left(H\right)=\sum_{k=1}^{2}c_{(ab)R}\left(H_k^{\pm}\right),$$

$$c_{(ba)R}=c_{(ab)R}\left(V\right)+c_{(ab)R}\left(H\right), c_{(ba)R}=\left\{c_{(ab)R}\left[a\leftrightarrow b\right]\right\}\times\frac{m_{e_a}}{m_{e_b}}. \tag{32}$$

Knowing $\Delta a_{e_a}\left(Y\right)\simeq 0$ as commented previously, the one-loop contributions from charged gauge bosons to $\Delta a_{e_a}$ are estimated approximately as follows [12],

$$\begin{aligned}\Delta a_{e_a}\left(V\right)&=\left[a_{e_a}\left(\mathrm{W}\right)-a_{e_a}^{SM}\left(\mathrm{W}\right)\right]\simeq-\frac{4m_{e_a}^2}{e}\left(\sum_{k=1}^{2}\mathrm{Re}\left[c_{(\mathrm{aa})R}\left(\mathrm{W}\right)-c_{(\mathrm{aa})R}^{SM}\left(\mathrm{W}\right)\right]\right)\\&=\frac{g^2m_{e_a}^2}{8\pi^2m_{\mathrm{W}}^2}\left[\frac{5}{12}\left(\tilde{m}_D^*V_R^*\hat{k}^{-2}V_R^T\tilde{m}_D\right)_{aa}-\sum_{e=1}^{3}\left|\left(\tilde{m}_D^*V_R^*\hat{k}^{-1}\right)_{ae}\right|^2F_V\left(x'_{\mathrm{W},e}\right)\right].\end{aligned} \tag{33}$$

The Eq. (33) results in a clearly consequence that $\Delta a_{e_a}\left(V\right)\simeq k_e^{-2}\sim\mathcal{O}\left(10^{-2}\right)$, because the condition $k_e\gg 1$ for all $e=1,2,3$ must be valid to guarantee the ISS relations. Other parameters are estimated as $\max\left[\left|\left(\tilde{m}_D\right)_{ab}\right|\right]\leq 1(5)$ for the NO (IO) scheme (in the numerical section), $\left|V_{ab}^R\right|\leq 1$ and $\left|F_V\left(x\right)\right|\leq 1/4$. For muon, $\frac{g^2m_\mu^2}{8\pi^2m_W^2}\simeq 9.2\times10^{-9}$, hence $\left|\Delta a_\mu\left(V\right)\right|\leq 9.2\times10^{-9}\times k_e^{-2}\times\left(5/12+1/4\right)\leq\mathcal{O}\left(10^{-11}\right)$ for the NO scheme. This conclusion is enough to explain the numerical results reported in Ref. [6]. Then, we can use the following formulas:

$$\Delta a_\mu\equiv\Delta a_{e_a}^{331ISS}=-\frac{4m_{e_a}^2}{e}\mathrm{Re}\left[c_{(aa)R}\right]\simeq a_{e_a}\left(H\right)=--\frac{4m_{e_a}^2}{e}\mathrm{Re}\left[c_{(aa)R}\left(H\right)\right], \tag{34}$$

where $\Delta a_\mu$ is considered as new physics predicted by the 3-3-1ISS in this work. It will be used to compare with the experimental data in the following numerical investigation. The more general formulas $c_{(ab)R}=c_{(ab)R}\left(W\right)+c_{(ab)R}\left(H\right)$ will be used in the numerical investigation for both IO and NO scheme.

The branching ratios of the cLFV decays considered in this work are [8, 12, 13]

$$Br_{(e_b\to e_a\gamma)}\simeq\frac{48\pi^2}{G_F^2}\left(\left|c_{(ab)R}\right|^2+\left|c_{(ba)R}\right|^2\right)Br_{(e_b\to e_a\bar{\nu}_a\nu_b)}, \tag{35}$$

where $G_F=g^2/\left(4\sqrt{2}m_{\mathrm{W}}^2\right)$. In the following numerical discussion, we will collect only points satisfying all cLFV upper constraints (3). Using the approximate formulas of charged Higgs couplings given in Eq. (27), we can confirm the results of $\Delta a_\mu$ are consistent with those reported in Ref. [6]. We will also investigate the numerical values of $\Delta a_e$ and compare with the latest experimental results in both NO and IO schemes of neutrino oscillation data.

## 4. Results and discussion

In this section, we will firstly consider the NO scheme of the neutrino data given in [14], and check the numerical results for $\Delta a_\mu$ given in Ref. [6] before investigating $\Delta a_e$. The standard form of the lepton mixing matrix $U_{PMNS}$ is the function of three angles $\theta_{ij}$, one Dirac phase δ and two Majorana phases $\alpha_1$ and $\alpha_2$. We will use the same best-fit point chosen in Ref. [6] for the NO scheme, $m_{n_1} < m_{n_2} < m_{n_3}$, as $s_{12}^2 = 0.32,\ s_{23}^2 = 0.547,\ s_{13}^2 = 0.0216,\ \delta = 180$ [Deg], $\Delta m_{21}^2 = 7.55\times10^{-5}$ [eV$^2$] and $\Delta m_{32}^2 = 2.424\times10^{-3}$ [eV$^2$] where $\Delta m_{ij}^2 \equiv m_{n_j}^2 - m_{n_i}^2, s_{ij} \equiv \sin\theta_{ij}$, and $c_{ij} \equiv \cos\theta_{ij} = \sqrt{1-s_{ij}^2}$. For the IO scheme, $m_{n_3} < m_{n_2} < m_{n_1}$, we choose a benchmark with best-fit points $s_{12}^2 = 0.32,\ s_{23}^2 = 0.547,\ s_{13}^2 = 0.0216,\ \Delta m_{21}^2 = 7.55\times10^{-5}$ [eV$^2$], and $\Delta m_{32}^2 = -2.509\times10^{-3}$ [eV$^2$]. For simplicity, in the IO case, the Dirac phase is chosen close to $\delta = 354 \simeq 360$ [Deg.], so that $\tilde{m}_D$ is real. Now $\tilde{m}_D$ in the NO and IO schemes are fixed as follows:

$$\tilde{m}_D(NO) = \begin{pmatrix} 0 & 0.613 & 0.357 \\ -0.613 & 0 & 1 \\ -0.357 & -1 & \end{pmatrix},\ \tilde{m}_D(IO) = \begin{pmatrix} 0 & 4.346 & -4.933 \\ 4.346 & 0 & 1 \\ -4.933 & -1 & \end{pmatrix}. \quad (36)$$

The IO scheme has the upper perturbative bound as $z_0 \le 1223/4.933 \simeq 247.9$ GeV, which is much smaller than the one that $z_0 \le 1223$ GeV for the NO scheme. In contrast, all $|\tilde{m}_D(NO)| \le 1$ while $|\tilde{m}_D(IO)| \ge 1$, which lead to different predictions of large $\Delta a_{e_a}$ in the two cases.

In both IO and NO neutrino mass spectrum, $V_R$ can be parameterized in the following form:

$$V_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^r & s_{23}^r \\ 0 & -s_{23}^r & c_{23}^r \end{pmatrix} \begin{pmatrix} c_{13}^r & 0 & s_{13}^r e^{-i\delta^r} \\ 0 & 1 & 0 \\ -s_{13}^r e^{i\delta} & 0 & c_{13}^r \end{pmatrix} \begin{pmatrix} c_{12}^r & s_{12}^r & 0 \\ -s_{12}^r & c_{12}^r & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (37)$$

where $s_{ij}^r \equiv \sin\theta_{ij}^r, c_{ij}^r \equiv \cos\theta_{ij}^r = \sqrt{1-\left(s_{ij}^r\right)^2}$, and $\delta^r = \pi$ or $2\pi$ corresponding to the NO and IO schemes. Other free parameters are chosen in the following ranges:

$$k_{1,2,3} \ge 5,\ m_{H_k^\pm} \ge 0.6 \text{ TeV},\ 0.5 \le t_\beta \le 80. \quad (38)$$

The numerical results for the NO scheme of neutrino data are shown in Fig. 1, where all collected pointed satisfy simultaneously all upper cLFV constraints given in Eq. (3) and the lower bound $\Delta a_\mu > 8.8\times10^{-10}$. While values of $\Delta a_e$ are consequences of these constraints. We can see that $\Delta a_e$ depends strongly on $\Delta a_\mu$. The maximal values of $\Delta a_\mu$ is $\max\left[\Delta a_\mu\right] \simeq 113.44\times10^{-11}$ which is slightly larger than the value 108.5 × $10^{-11}$ reported in Ref. [6]. We can also see that $\max\left[\Delta a_e\right] \simeq 2.52\times10^{-14}$, which is much smaller than the $1\sigma$ values of experimental data of the order $\mathcal{O}\left(10^{-13}\right)$ given in Eq. (2). The $\max \Delta a_\mu$ corresponds to large $Br(\tau \to \mu\gamma) \sim \mathcal{O}\left(10^{-8}\right)$, which is very close to the experimental upper bound given in Eq. (3). In contrast, two other decays $\mu \to e\gamma$ and $\tau \to e\gamma$ values rather smaller that the experimental upper bounds. This conclusion is consistent with the one obtained from the previous numerical investigation in Ref. [6].

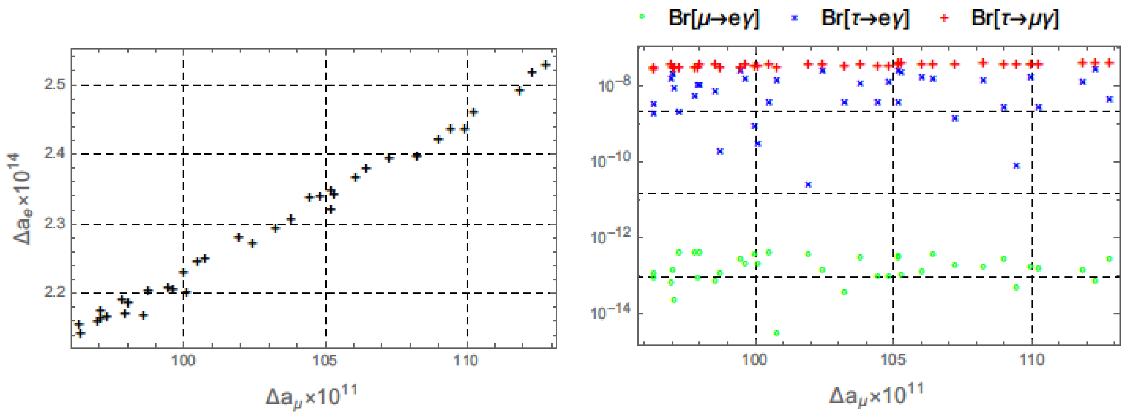


**Figure 1.** The correlations of $\Delta a_\mu$ vs $\Delta a_e$ (left panel) and Br( $e_b \to e_a\gamma$ ) (right panel) in the region satisfying all cLFV constraints and $\Delta a_\mu > 8.8\times10^{-10}$ in the NO scheme.

We have also scanned the parameter space of the 3-3-1ISS model to answer the question that did not consider previously: are there any regions of the parameter space that allow large $\Delta a_e$ close to the 1σ experimental data given in Eq. (2). The numerical results to look for large $\Delta a_e$ in the NO scheme are shown in Fig. 2, where the collected points satisfy simultaneously all upper cLFV constraints given in Eq. (3) and $\Delta a_e > 1.9\times10^{-14}$.

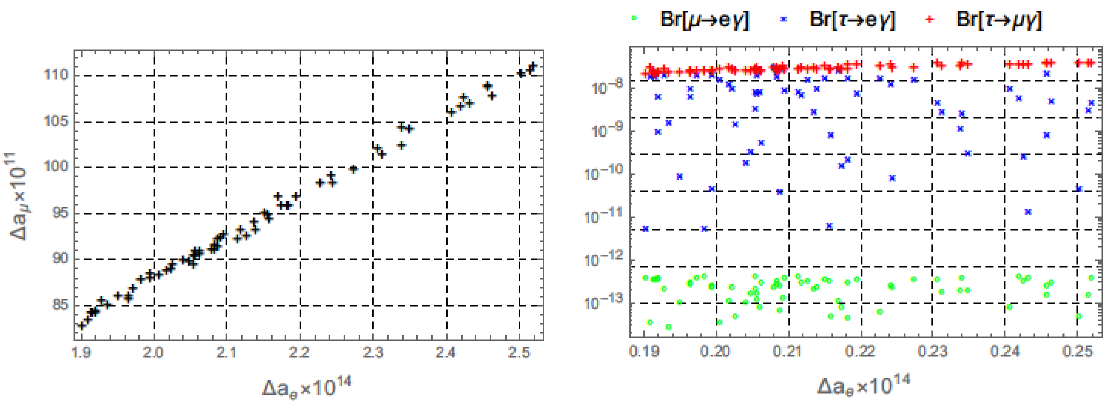


**Figure 2.** The correlations of $\Delta a_\mu$ vs $\Delta a_e$ (left panel) and Br( $e_b \to e_a\gamma$ ) (right panel) in the region satisfying all cLFV constraints and $\Delta a_e > 1.9\times10^{-14}$ in the NO scheme

The values of $\Delta a_\mu$ result from these constraints. We again obtain the results consistent with those illustrated in Fig. 2. Namely, $\max[\Delta a_e] \simeq 2.5\times10^{-14}$ results in $\max[\Delta a_\mu] \simeq 110\times10^{-11}$ and both of them are strictly constrained by the experiments of $Br(\tau \to \mu\gamma) < 4.2\times10^{-8}$.

We also scanned the parameter space to find the regions giving largest Δaμ in the IO scheme, see illustrations shown in Fig. 3, where all collected points satisfying cLFV constraints and $\Delta a_\mu > 5.5\times10^{-10}$. It turns out that $\max\left[\Delta a_{e_a}\right]$ is much smaller than those obtained in the NO scheme. Namely, the left panel of Fig. 3 gives $\max\left[\Delta a_\mu\right] \le 59\times10^{-11}$ and $\max[\Delta a_e] \le 1.4\times10^{-14}$. These two maximal values are also originated from the experimental constraint $Br(\tau \to \mu\gamma) < 4.2\times10^{-8}$, as seen in the right panel of Fig. 3. The numerical results show a very special property that $\Delta a_{\mu,e}$ are constrained strongly by $Br(e_a \to e_b\gamma)$, which can be explained qualitatively using the analytic formulas given in Eq. (31). One-loop contributions from the two singly charged Higgs bosons consists of many dominant parts depending on $\tilde{m}_D^* \tilde{m}_D$, which is nearly unchanged as given in Eq. (36).

Because $\Delta a_e \sim c_{(aa)R}$ relating with $\left[\tilde{m}_D^* \tilde{m}_D\right]_{aa}$, while $Br(e_a \to e_b\gamma) \sim \left(|c_{(ab)R}|^2 + |c_{(ba)R}|^2\right)$ relating with $\left[\tilde{m}_D^* \tilde{m}_D\right]_{ab}$ and $\left[\tilde{m}_D^* \tilde{m}_D\right]_{ba}$. therefore, large $\Delta a_{e_a} \sim c_{(aa)R}$ and small $Br(e_a \to e_b\gamma)$ require the strong destructive correlations between different terms appearing in $c_{(ab)R}$ and $c_{(ba)R}$. This cancellations are not enough to allow large $a_{\mu,e}$ to reach the experimental data.

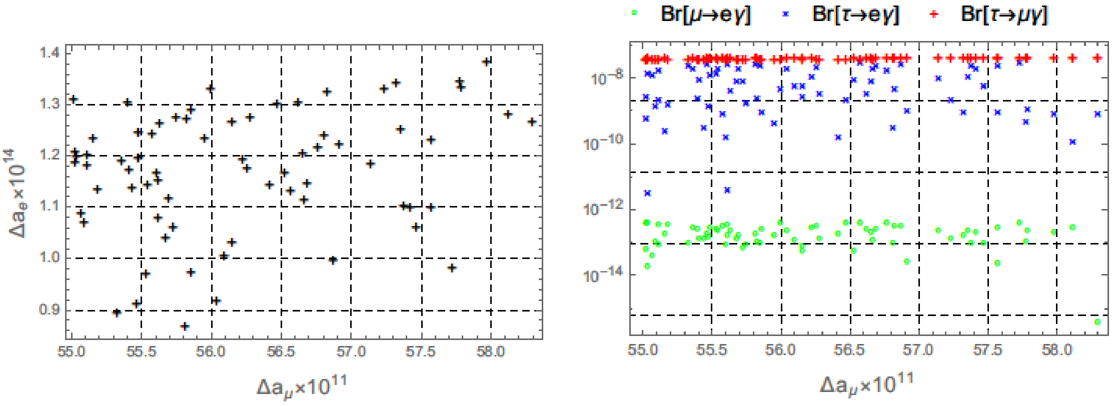


**Figure 3.** The correlations of $\Delta a_\mu$ vs $\Delta a_e$ (left panel) and Br( $e_b \to e_a\gamma$ ) (right panel) in the region satisfying all cLFV constraints and $\Delta a_\mu > 5.5\times10^{-10}$ in the IO scheme.

## 5. Conclusions

We have derived the analytic formulas up to one-loop contributions for calculating $(g-2)_{e,\mu}$ anomalies and $Br(e_a \to e_b\gamma)$ predicted by the 3-3-1ISS model. Using these formulas, we reproduce all of the numerical results reported previously for $\Delta a_\mu$. We can also explain clearly these numerical results. Investigating both $\Delta a_e$ and $\Delta a_\mu$ in the wide ranges of the parameter space we have derived the largest values of $\Delta a_e$ ( $\Delta a_\mu$ ) are around 2.5 × $10^{-14}$ (1.13 × $10^{-9}$) and 1.4 × $10^{-14}$ (0.58 × $10^{-9}$) for the NO and IO schemes, respectively. Therefore, the 3-3-1ISS model adding only neutral lepton gauge singlets cannot explain the recent $(g-2)_{e,\mu}$ data. We would like emphasize that the analytic formulas and the way to derive them which presented in this work will be very useful for further studies on other extensions of the 3-3-1 models constructed to accommodate all experimental data on the lepton sector.

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