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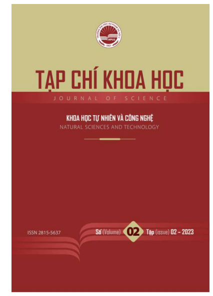

Article type: Research article

Decay of charged Higgs boson $H^{\pm} \to W^{\pm}\gamma$ in the 3-3-1 model with neutral leptons

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Abstract

In the model 3-3-1 with neutral leptons, we used the condition imposed on the Higgs potential to derive mass spectra and physics states of neutral and charged Higgs bosons related to $H^{\pm} \to W^{\pm}\gamma$. We also calculate all couplings and give Feynman diagrams. Most importantly, we give analytical results for the contributing components and the total amplitude of $H^{\pm} \to W^{\pm}\gamma$.

Keywords: Extensions of electroweak Higgs sector, Electroweak radiative corrections, Charged gauge boson, Quark mass and mixing, 2HDM.

1. Introduction

After the discovery of the Higgs boson in 2012, the scalar sector continued to be of naturally interest as a seamless connection. In particular, they are data related to the charged Higgs boson.

The decay channels of the charged Higgs boson are also of particular interest, usually in two directions: i) the charged Higgs boson decays into two bosons ($H^{\pm} \to W^{\pm} + h^0, Z, \gamma$), ii) the charged Higgs boson decays into two fermions ($H^{+} \to t\bar{b}$).

Based on the decay $H^{\pm} \to W^{\pm}Z$, the Georgi-Machacek Higgs Triplet Model (the GM model) predicted the mass of the charged Higgs to be in the range of $240\ GeV < m_{H^{\pm}} < 700\ GeV$ at the LHC's active energy scale of 8TeV (LHC@8 TeV). However, experimental data have shown that there is no signal to confirm the above results [1]. This publication also shows that the mass of the charged Higgs boson is outside the range of $200\ GeV < m_{H^{\pm}} < 1000\ GeV$ with 95% CL (confidence level).

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Besides, the experiment also shows the existence of the branching ratio of $H^{\pm} \to W^{\pm}Z$ even though it is very small. Clearly, the search results depend on the active energy of the LHC. The GM model is still working, but with the $H^{+} \to t\overline{b}$ decay channel at LHC@13 TeV, the mass prediction of charged Higgs boson is about 250 GeV with 95% CL and it is predicted that the signal of $H^{\pm} \to W^{\pm}h$ can be obtained at higher energy [2].

Some other studies based on $H^{\pm} \to W^{\pm}Z, W^{\pm}\gamma$ decays have also shown that the mass of the charged Higgs boson takes on a value of about 600 GeV [3] or 1000 GeV [4,5]. These are reliable results at LHC@ 13 TeV.

Recently, the search for the mass and interactions of the charged Higgs boson has remained active. In the framework of GM model, at LHC@14 TeV, one studies the decay $H^{\pm} \to W^{\pm}\gamma$ and the results have shown that $m_{H^{\pm}} \geq 130\ GeV$ [6].

It is necessary to expand the study of $H^{\pm} \to W^{\pm}\gamma$ decay in other models and promises to give many interesting results. For example, the classes of 3-3-1 models with $\beta = -\frac{1}{\sqrt{3}}$, explained very well the experimental results of the LFV decay channels of the charged lepton [7,8] or the SM-like Higgs boson [9,10,11,12]. In particular, some models with neutral leptons can also analyze the contribution of the charged Higgs boson in rare decay processes [13,14].

In the framework of this paper, we will study the $H^{\pm} \to W^{\pm}\gamma$ decay in model 3-3-1 with neutral leptons, in order to show the interactions of the related charged Higgs boson and give analytical results for the contribution components as well as the total amplitude.

The paper is organized as follows. In the next section, we review the model, give masses spectrum of gauge and Higgs bosons relate to $H^{\pm} \to W^{\pm}\gamma$. We give all couplings in Section 3 and analytic formulas in Section 4. Conclusions are in Section 5.

2. Review of the model

The 3-3-1 model with neutral leptons is structured based on gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ with parameter $\beta = -\frac{1}{\sqrt{3}}$. Therefore, the model's charge operator has the form: $Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X$ where $T_{3,8}$ are diagonal $SU(3)_L$ generators and X is the new charge of the group $U(1)_X$. The biggest difference of this model from other 3-3-1 model classes is neutral particles lying bottom of triplet leptons and its right-handed component is singlet of $SU(3)_L$.

Therefore, fermions consist of leptons and quarks arranged as follows [15]:

$$\Psi'_{aL} = \begin{pmatrix} \nu'_a \\ e'_a \\ N'_a \end{pmatrix}_L \sim \left(1,3,-1/3\right), e'_{aR} \sim \left(1,1,-1\right), N'_{aR} \sim \left(1,1,0\right), \tag{1}$$

$$Q'_{iL} = \begin{pmatrix} d'_i \\ -u'_i \\ D'_i \end{pmatrix}_L \sim \left(3, \overline{3}, 0\right),\ u'_{iR} \sim \left(3,1,2/3\right), d'_{iR} \sim \left(3,1,-1/3\right), D'_{iR} \sim \left(3,1,-1/3\right), \tag{2}$$

$$Q'_{3L} = \begin{pmatrix} u'_3 \\ d'_3 \\ U'_3 \end{pmatrix}_L \sim \left(3,3,1/3\right),\ u'_{3R} \sim \left(3,1,2/3\right), d'_{3R} \sim \left(3,1,-1/3\right), U'_{3R} \sim \left(3,1,2/3\right), \tag{3}$$

where $a = 1,2,3$ and $i = 1,2$ are generation indexes. The left-handed components are assigned to triplets and the right-handed components are assigned to singlets of the $SU(3)_L$ group. The three quantum numbers in parentheses after each multiplet represent the charge of $SU(3)_C$, $SU(3)_L$, $U(1)_X$, respectively.

To generate masses for the particles, one give three scalar triplets. Where, the two triplets η and χ have the same quantum numbers.

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^0 \end{pmatrix} \sim (1,3,-\frac{1}{3}),\ \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho'^+ \end{pmatrix} \sim (1,3,\frac{2}{3}),\ \chi = \begin{pmatrix} \chi'^0 \\ \chi^- \\ \chi^0 \end{pmatrix} \sim (1,3,-\frac{1}{3}), \tag{4}$$

The vacuum expectation values (VEVs) are chosen to satisfy the following conditions: i) generate mass for the particles in the model ii) avoid large violations of lepton number occurring at neutral currents iii) free parameters in the model are minimal. Thus VEVs are introduced [8,14]:

$$\begin{aligned} &\eta'^0 = \frac{S'_2 + iA'_2}{\sqrt{2}}, \chi'^0 = \frac{S'_3 + iA'_3}{\sqrt{2}}, \rho^0 = \frac{1}{\sqrt{2}}\left(v_1 + S_1 + iA_1\right), \\ &\eta^0 = \frac{1}{\sqrt{2}}\left(v_2 + S_2 + iA_2\right), \chi^0 = \frac{1}{\sqrt{2}}\left(v_3 + S_3 + iA_3\right), \end{aligned} \tag{5}$$

The Higgs potential is given in its most common form. Here only the terms that preserve the lepton number are kept, the rest are ignored because there are very small accompanying coefficients [16]. For the convenience of later calculations, the constant f is chosen to be dimensionless.

$$\begin{aligned} &V\left(\eta,\rho,\chi\right) = \mu_1^2\eta^2 + \mu_2^2\rho^2 + \mu_3^2\chi^2 + \lambda_1\eta^4 + \lambda_2\rho^4 + \lambda_3\chi^4 \\ &+\lambda_{12}\left(\eta^+\eta\right)\left(\rho^+\rho\right) + \lambda_{13}\left(\eta^+\eta\right)\left(\chi^+\chi\right) + \lambda_{23}\left(\rho^+\rho\right)\left(\chi^+\chi\right) \\ &+\tilde{\lambda}_{12}\left(\eta^+\rho\right)\left(\rho^+\eta\right) + \tilde{\lambda}_{13}\left(\eta^+\chi\right)\left(\chi^+\eta\right) + \tilde{\lambda}_{23}\left(\rho^+\chi\right)\left(\chi^+\rho\right) \\ &+\sqrt{2}fv_3\left(\epsilon^{ijk}\ \eta_i\rho_j\chi_k + h.c\right). \end{aligned} \tag{6}$$

From the minimum condition of the Higgs potential, we will get the mass spectrum of the Higgs bosons in the model. This process is done in the same way as in Refs.[13,14,15].

This model has two charged Higgs bosons, the mass and physical state are given as follows:

$$m_{H_1\pm}^2 = v_1^2\left(\frac{1}{t_{12}^2}+1\right)\left(\frac{\tilde{\lambda}_{12}}{2}+\frac{ft_{12}}{t_{13}^2}\right); m_{H_2\pm}^2 = v_1^2\left(\frac{1}{t_{13}^2}+1\right)\left(\frac{\tilde{\lambda}_{23}}{2}+\frac{f}{t_{12}}\right), \quad (7)$$

$$\begin{pmatrix}\rho^\pm\\ \eta^\pm\end{pmatrix} = \begin{pmatrix}-c_{12} & s_{12}\\ s_{12} & c_{12}\end{pmatrix}\begin{pmatrix}G_W^\pm\\ H_1^\pm\end{pmatrix}, \begin{pmatrix}\rho'^\pm\\ \chi^\pm\end{pmatrix} = \begin{pmatrix}-s_{13} & c_{13}\\ c_{13} & s_{13}\end{pmatrix}\begin{pmatrix}G_V^\pm\\ H_2^\pm\end{pmatrix}. \quad (8)$$

As shown in Ref.[14], $H_2^\pm$ has mass proportional to symmetry breaking scale of $SU(3)_L$ group ($\sim v_3$) but does not interact with $\mathrm{W}^\mp$ and Z. The remaining ($H_1^\pm$) participate in $H_1^\pm \mathrm{W}^\mp Z$ coupling so its mass between $600[GeV]$ and $1000[GeV]$ [3,4]. This is the range of predicted values that can be found in the near future by experiment. Therefore, we will study $H_1^\pm \to W^\pm\gamma$ decay in the framework of this paper.

This model has four neutral Higgs bosons. However, h_4^0 is assumed not to interact with particles like the standard model [14], and h_3^0 does not pair with $\mathrm{W}^\mp$ and $H_1^\mp$ [14]. Therefore, only $h_{1,2}^0$ actually contributes to $H_1^\pm \to W^\pm\gamma$ decay.

By applying the condition imposed on the constant f ($f = \lambda_{13}t_{12} = \frac{\lambda_{23}}{t_{12}}$), it is shown that the masses of the neutral Higgs bosons $h_{1,2}^0$ as in Refs. [13,14] are:

$$\begin{aligned}
m_{h_1^0}^2 &= M_{11}^2\cos^2\delta + M_{22}^2\sin^2\delta - M_{12}^2\sin 2\delta,\\
m_{h_2^0}^2 &= M_{11}^2\sin^2\delta + M_{22}^2\cos^2\delta + M_{12}^2\sin 2\delta,\\
M_{11}^2 &= 2\left(s_{12}^4\lambda_1 + c_{12}^4\lambda_2 + s_{12}^2c_{12}^2\lambda_{12}\right)v^2 = \mathcal{O}\left(v^2\right),\\
M_{12}^2 &= \left[-\lambda_1 s_{12}^2 + \lambda_2 c_{12}^2 + \lambda_{12}\left(s_{12}^2 - c_{12}^2\right)\right]s_{12}c_{12}v^2 = \mathcal{O}\left(v^2\right),\\
M_{22}^2 &= 2s_{12}^2c_{12}^2\left[\lambda_1 + \lambda_2 - \lambda_{12}\right]v^2 + \frac{\lambda_{13}v_3^2}{c_{12}^2},
\end{aligned} \quad (9)$$

Where $\tan 2\delta = \frac{2M_{12}^2}{M_{22}^2 - M_{11}^2} \sim \mathcal{O}\left(\frac{v^2}{v_3^2}\right)$ and $v^2 = v_1^2 + v_2^2$ (10)

In above formulas, we use denotations: $t_{12} = \tan\beta_{12} = \frac{v_2}{v_1}, t_{13} = \tan\beta_{13} = \frac{v_1}{v_3}, t_{23} = \tan\beta_{23} = \frac{v_2}{v_3}$ and $s_{\mathrm{ij}} = \sin\beta_{\mathrm{ij}}, c_{\mathrm{ij}} = \cos\beta_{\mathrm{ij}}$.

Based on δ - parameter characteristic for all 2HDM [15,16], one gives relationship between the physical state and the original basic

$$\begin{pmatrix}S_1\\ S_2\end{pmatrix} = \begin{pmatrix}-s_\alpha & c_\alpha\\ c_\alpha & s_\alpha\end{pmatrix}\begin{pmatrix}h_1^0\\ h_2^0\end{pmatrix} \text{ and } \alpha = \beta_{12} - \frac{\pi}{2} + \delta \quad (11)$$

where h_1^0 is the lightest and is identical to the Higgs boson of the standard model (SM-like

Higgs boson).

The gauge bosons in the model are given based on the covariance derivative

$$D_\mu \equiv \partial_\mu - ig_3 W_\mu^\alpha T^\alpha - g_1 T^9 XX_\mu, \tag{12}$$

where i is the complex number and $T^\alpha, \alpha = \overline{1,8}$ are the generators of the $SU(3)_L$ group , $T^9 = \frac{1}{\sqrt{6}}\mathbb{1}$.

Only photon of neutral gauge bosons involve to $H_1^\pm \to W^\pm \gamma$, the remaining should be ignored. We focus the charged part is

$$W_\mu^a T^a = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ & U_\mu^0 \\ W_\mu^- & 0 & V_\mu^- \\ U_\mu^{0*} & V_\mu^+ & 0 \end{pmatrix} \tag{13}$$

Where $\mathrm{W}_\mu^\pm = \frac{\mathrm{W}_\mu^1 \mp i\mathrm{W}_\mu^2}{\sqrt{2}}$; $V_\mu^\pm = \frac{\mathrm{W}_\mu^6 \pm i\mathrm{W}_\mu^7}{\sqrt{2}}$; $U_\mu^{0*,0} = \frac{\mathrm{W}_\mu^4 \pm i\mathrm{W}_\mu^5}{\sqrt{2}}$ and their masses

$$m_W^2 = \frac{g^2}{4}\left(v_1^2 + v_2^2\right), m_U^2 = \frac{g^2}{4}\left(v_2^2 + v_3^2\right), m_V^2 = \frac{g^2}{4}\left(v_1^2 + v_3^2\right) \tag{14}$$

We reiterate that $V^\pm$ and $U^{0,0*}$ do not interact with $H_1^\pm$ [14] so they do not contribute to $H_1^\pm \to W^\pm \gamma$.

3. Couplings related to charged Higgs bosons decay

In this section we will introduce couplings related to $H_1^\pm \to W^\pm \gamma$ decay, they include: couplings of quarks from Lagrangian Yukawa, couplings of scalars and couplings relate to gauge bosons.

3.1. Couplings of quarks from Lagrangian Yukawa

Lagrangian Yukawa for quarks [17,18].

$$\begin{aligned} -\mathcal{L}_{quark}^{Yukawa} &= h_{ia}^d \overline{Q'}_{iL} \eta^* d'_{aR} + h_{3a}^u \overline{Q'}_{3L} \eta u'_{aR} + h_{3a}^d \overline{Q'}_{3L} \rho d'_{aR} \\ &+ h_{ia}^u \overline{Q'}_{iL} \rho^* u'_{aR} + h_{33}^U \overline{Q'}_{3L} \chi U_{3R} + h_{ii'}^D \overline{Q'}_{iL} \chi^* D_{i'R} + h.c., \end{aligned} \tag{15}$$

as previously denoted, $a = 1,2,3$ is the generation index and $i, i' = 1,2$.

We assume that M^u and M^d are matrices mixing the masses of the u-quarks and d-quarks, which results in the physical mass of the ordinary quarks being:

$diag(m_{u_1}, m_{u_2}, m_{u_3}) = V_L^u M^u V_R^{u\dagger}, diag(m_{d_1}, m_{d_2}, m_{d_3}) = V_L^d M^d V_R^{d\dagger}$ where V_L^u and V_L^d are related to each other through the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $V_{CKM} = V_L^u V_L^{d\dagger}$

It mean that, $m_{u_a} \equiv \left(V_L^u\right)_{ab} \frac{h_{bc}^u v_s}{\sqrt{2}} \left(V_R^{u\dagger}\right)_{ca}$, with $\begin{cases} b = 1,2 \to s = 1 \\ b = 3 \to s = 2 \end{cases}$ (16)

and $m_{d_a} \equiv \left(V_L^d\right)_{ab} \frac{h_{bc}^d v_s}{\sqrt{2}} \left(V_R^{d\dagger}\right)_{ca}$, with $\begin{cases} b = 1,2 \rightarrow s = 2 \\ b = 3 \rightarrow s = 1 \end{cases}$ (17)

We can come up with the following alternative:

$$h_{bc}^d \equiv \frac{\sqrt{2} m_{d_a}}{v_s} \left(V_L^{d\dagger}\right)_{ab} \left(V_R^d\right)_{ca},\ h_{bc}^u \equiv \frac{\sqrt{2} m_{u_a}}{v_s} \left(V_L^{u\dagger}\right)_{ab} \left(V_R^u\right)_{ca} \quad (18)$$

As a result, the relationship between the initial states and the physical states of the quarks are:

$$u_i^{'} = \left(V_R^{u\dagger}\right)_{\mathrm{ij}} u_j,\ d_i^{'} = \left(V_R^{d\dagger}\right)_{\mathrm{ij}} d_j, \overline{u}_i^{'} = \left(V_L^u\right)_{\mathrm{ij}} \overline{u}_j,\ \overline{d}_i^{'} = \left(V_L^d\right)_{\mathrm{ij}} \overline{d}_j \quad (19)$$

We can rewrite eq. (15) as:

$$\begin{aligned} -\mathcal{L}_{quark}^{Yukawa} \supset & -h_{ia}^d \overline{u}_{bL} \left(V_L^u\right)_{ib} c_{12} H_1^+ \left(V_R^{d\dagger}\right)_{ac} d_{cR} + h_{3a}^u \overline{d}_{bL} \left(V_L^d\right)_{3b} c_{12} H_1^- \left(V_R^{u\dagger}\right)_{ac} u_{cR} \\ & + h_{3a}^d \overline{u}_{bL} \left(V_L^u\right)_{3b} s_{12} H_1^+ \left(V_R^{d\dagger}\right)_{ac} d_{cR} + h_{ia}^u \overline{d}_{bL} \left(V_L^d\right)_{ib} s_{12} H_1^- \left(V_R^{u\dagger}\right)_{ac} u_{cR} \\ & + h_{3a}^d \overline{U}_{3L} c_{13} H_2^+ \left(V_R^{d\dagger}\right)_{ac} d_{cR} + h_{ia}^u \overline{D}_{iL} c_{13} H_2^- \left(V_R^{u\dagger}\right)_{ac} u_{cR} \\ & + h_{33}^U \overline{d}_{cL} \left(V_L^d\right)_{3c} s_{13} H_2^- U_{3R} - h_{ii'}^D \overline{u}_{cL} \left(V_L^u\right)_{ic} s_{13} H_2^+ D_{i'R} + h.c, \end{aligned} \quad (20)$$

We find out that exotic quarks only interact with $H_2^{\pm}$, thus, the corresponding coefficients of $H_1^{\pm}$ are:

$$g\left(H_1^+ \overline{u}_b d_c\right) = \sqrt{2}\left[\left(-\frac{m_{u_b}}{\sqrt{v_1^2 + v_2^2}} (V_{CKM})_{bc} \frac{1}{t_{12}}\right) P_L + \left((V_{CKM})_{bc} \frac{m_{d_c}}{\sqrt{v_1^2 + v_2^2}} (-\delta_{1i} t_{12} + \delta_{2i} \frac{1}{t_{12}})\right) P_R\right]$$

$$g\left(H_1^- u_b \overline{d}_c\right) = -\sqrt{2}\left[\left(-\frac{m_{d_b}}{\sqrt{v_1^2 + v_2^2}} (V_{CKM}^\dagger)_{bc} (-\delta_{1i} t_{12} + \delta_{2i} \frac{1}{t_{12}})\right) P_L + \left((V_{CKM}^\dagger)_{bc} \frac{m_{u_c}}{\sqrt{v_1^2 + v_2^2}} \frac{1}{t_{12}}\right) P_R\right] \quad (21)$$

Couplings of quarks with gauge bosons will be given later.

3.2. Couplings of scalars

The couplings of Higgs bosons is given from the Higgs potential (Eq.6), but we are now only interested in third-order interactions. Specifically, it is the interaction of two charged Higgs bosons with a neutral Higgs boson. As noted above, the neutral Higgs bosons in this model have only $h_{1,2}^0$ interactions with $H_1^{\pm}$. The detailed calculations in Ref.[14] also show that the $H_1^+ H_2^- h_1^0$, $H_1^+ H_2^- h_2^0$ vertices are suppressed. Therefore, we only interest two vertices follow:

$$g\left(H_1^+ H_1^- h_1^0\right) = iv[\left(s_{12}^3 c_\alpha - c_{12}^3 s_\alpha\right)\left(\lambda_{12} + \tilde{\lambda}_{12}\right) - s_{12}^2 c_{12} s_\alpha \left(2\lambda_2 + \tilde{\lambda}_{12}\right) + c_{12}^2 s_{12} c_\alpha \left(2\lambda_1 + \tilde{\lambda}_{12}\right)]$$

$$g\left(H_1^+ H_1^- h_2^0\right) = iv[\left(s_{12}^3 s_\alpha + c_{12}^3 c_\alpha\right)\left(\lambda_{12} + \tilde{\lambda}_{12}\right) + s_{12}^2 c_{12} c_\alpha \left(2\lambda_2 + \tilde{\lambda}_{12}\right) + c_{12}^2 s_{12} s_\alpha \left(2\lambda_1 + \tilde{\lambda}_{12}\right)]$$

(22)

3.3. Couplings relate to gauge bosons

The couplings of the Higgs boson and the gauge bosons are derived from the kinetic terms of the scalar fields.

$$\mathcal{L}_{\phi}^{kin} = \sum_{\phi=\eta,\rho,\chi} \left(D^{\mu}\phi\right)^{\dagger}\left(D_{\mu}\phi\right) \tag{23}$$

Expanding this term, we get the interactions between the Higgs bosons and the gauge bosons. This approach has been shown in ref.[19]. A special feature is the interaction between the three gauge bosons, their couplings given from the standard field strength tensor:

$$\begin{aligned} -\mathcal{L}^{Gauge} &= \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &\supset ie\{(\partial^{\alpha}\mathrm{W}^{+\beta})\mathrm{W}^{-\mu}\mathrm{A}^{\nu}(\mathrm{g}_{\alpha\mu}\mathrm{g}_{\beta\nu}\text{-}\mathrm{g}_{\alpha\nu}\mathrm{g}_{\beta\mu})+\mathrm{W}^{+\beta}(\partial^{\alpha}\mathrm{W}^{-\mu})\mathrm{A}^{\nu}(\mathrm{g}_{\alpha\nu}\mathrm{g}_{\beta\mu}\text{-}\mathrm{g}_{\alpha\beta}\mathrm{g}_{\mu\nu}) \\ &+\mathrm{W}^{+\beta}\mathrm{W}^{-\mu}(\partial^{\alpha}\mathrm{A}^{\nu}))(\mathrm{g}_{\alpha\beta}\mathrm{g}_{\mu\nu}\text{-}\mathrm{g}_{\alpha\mu}\mathrm{g}_{\beta\nu})\} \end{aligned} \tag{24}$$

Substituting the initial states by physical states, we obtain interactions of this type related to decay $H_1^{\pm} \to W^{\pm}\gamma$ as shown in Table 1.

Table 1: Couplings of gauge bosons related to decay $H_1^{\pm} \to W^{\pm}\gamma$

Vertex	Coupling	Vertex	Coupling
$g\left(H_1^{+}\mathrm{W}_{\mu}^{-}h_1^{0}\right)$	$-\frac{ig}{2}\left(c_{\alpha}s_{12}-s_{\alpha}c_{12}\right)\left(p_{h_1^0}-p_{H_1^+}\right)_{\mu}$	$g\left(H_1^{+}\mathrm{W}_{\mu}^{-}h_2^{0}\right)$	$-\frac{ig}{2}\left(c_{\alpha}s_{12}-s_{\alpha}c_{12}\right)\left(p_{h_1^0}-p_{H}\right.$
$\mathrm{g}\left(\mathrm{W}^{\mu+}\mathrm{W}_{\mu}^{-}h_1^{0}\right.$	$igm_{\mathrm{W}}\left(c_{\alpha}s_{12}-s_{\alpha}c_{12}\right)$	$\mathrm{g}\left(\mathrm{W}^{\mu+}\mathrm{W}_{\mu}^{-}h_2^{0}\right.$	$igm_{\mathrm{W}}\left(c_{\alpha}c_{12}+s_{\alpha}s_{12}\right)$
$g\left(H_1^{+}\mathrm{W}_{\mu}^{-}h_1^{0}\gamma\right.$	$\frac{g^2 s_W}{6}\left(c_{\alpha}c_{12}(1+2\sqrt{3})+s_{\alpha}s_{12}(4\sqrt{3}\right.$	$g\left(H_1^{+}\mathrm{W}_{\mu}^{-}h_2^{0}\gamma\right.$	$-\frac{ig}{2}\left(c_{\alpha}s_{12}-s_{\alpha}c_{12}\right)\left(p_{h_1^0}-p_{H}\right.$
$g\left(H_1^{+}H_1^{-}\gamma\right)$	$ie\left(p_{H_1^+}-p_{H_1^-}\right)$	$\mathrm{g}\left(\mathrm{W}_{\mu}^{+}\mathrm{W}^{\mu-}\gamma\right)$	ie

The last couplings relate to gauge bosons occur at quark sector. They are derived from the kinetic energy terms of the fermions.

$$\mathcal{L}_{q}^{kin} = \sum_{Q,q}\bar{Q}\gamma^{\mu}D_{\mu}Q \supset \sum_{q=u_a,d_a} eQ_q\bar{q}\gamma^{\mu}qA_{\mu} + \left(\sum_{u_a,d_a}\frac{eV_{CKM}}{s_W}\bar{u}\gamma^{\mu}P_L dW_{\mu}^{+} + h.c\right), \tag{25}$$

Thus, the relevant interactions given are:

$$g\left(\bar{q}qA_{\mu}\right) = iQ_q e\gamma^{\mu},\ g\left(\bar{u}d\mathrm{W}_{\mu}^{+}\right) = i\frac{e}{s_W}V_{CKM}\gamma^{\mu}P_L,\ g\left(u\bar{d}\mathrm{W}_{\mu}^{-}\right) = -i\frac{e}{s_W}V_{CKM}\gamma^{\mu}P_L \tag{26}$$

These interaction vertices are the basis for us to study decay $H_1^{\pm} \to W^{\pm}\gamma$.

4. Analytic formulas

Based on couplings shown in Sec.3, we give all Feynman diagrams of $H^{\pm} \to W^{\pm}\gamma$ at one-loop order in figure 1. To avoid unwanted interactions, we use the unitary gauge.

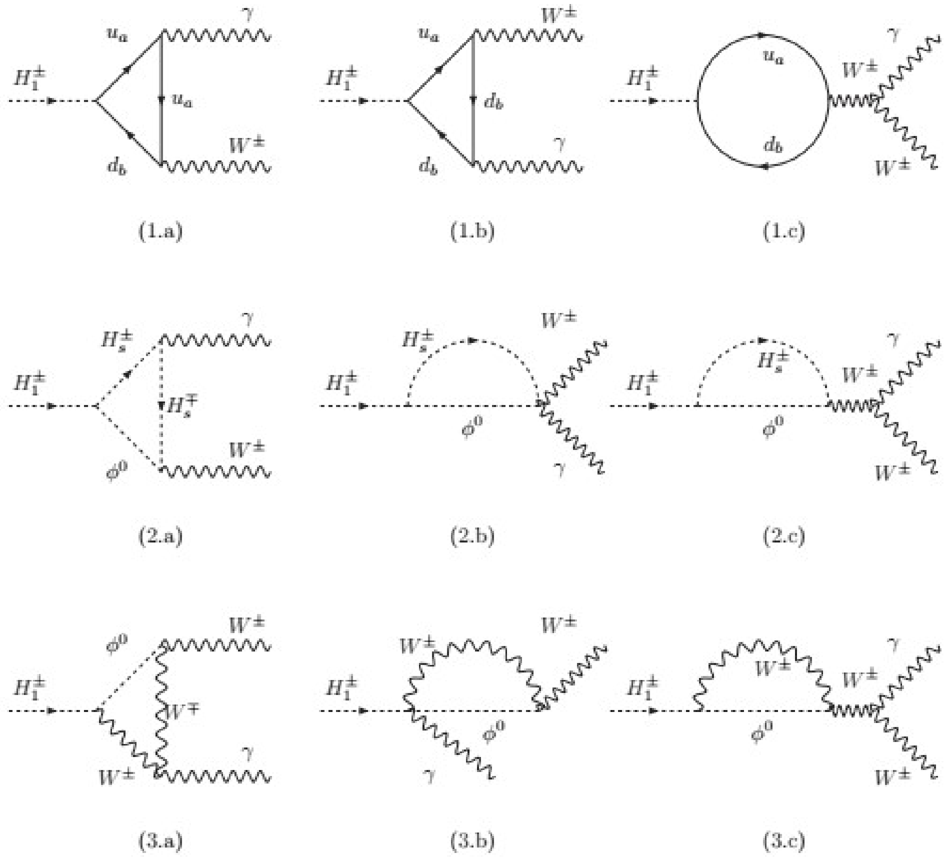

Fig 1: Feynman diagrams for $H^\pm \to W^\pm\gamma$ decay in 3-3-1 model with neutral lepton in unitary gauge. We use notations $H_s^\pm = H_{1,2}^\pm$ and $\phi^0 = h_{1,2}^0$

In Figure 1, the first row present one-loop diagrams with the contribution of quarks. Since the exotic quarks do not interact with $H_1^\pm$, only ordinary quarks as appear in SM, in the second row, we have the contribution of the Higgs bosons in loop. The neutral Higgs bosons are $h_{1,2}^0$. As pointed out in ref. [14], there is no third order interaction of three other particles among $h_{1,2}^0$ and the charged Higgs bosons ($H_1^\pm$, $H_2^\pm$), so $H_s^\pm = H_1^\pm$. In the third row , we have the contribution of the neutral Higgs bosons and the charged gauge bosons in the loop. Same as in the second line, we note $\phi^0 = h_{1,2}^0$. In particular, there are quaternary interaction between two Higgs bosons and two gauge bosons as shown in Table 1.

In general, this decay of the charged Higgs boson is written as:

$$H^+(k+q) \to \mathrm{W}^+(k)\gamma(q) \qquad (27)$$

Therefore, the amplitude of the decay is $M = \Gamma^{\mu\nu}\varepsilon_\mu^*(k)\varepsilon_\nu^*(q)$, where *k* and *q* are the momentum of the *W*-boson and the photon, respectively.

Based on Refs.[19, 20], we can give:

$$\Gamma^{\mu\nu} = (g^{\mu\nu} k.q - k^{\mu}.q^{\nu})S + i\varepsilon^{\mu\nu\alpha\beta} k_{\alpha} q_{\beta} \tilde{S} \tag{28}$$

where, $S = S^{(1)} + S^{(2)} + S^{(3)}$ is derived from the Feynman diagrams in Figure 1, with $S^{(1)}, S^{(2)}, S^{(3)}$ representing the first, second and third row contributions, respectively. Using the Ward-Takahashi identity $q_{\mu}\Gamma^{\mu\nu} = 0$, in case q is the photon's external momentum and the Ward-like identity $k_{\nu}\Gamma^{\mu\nu} = 0$ as given in Refs. [20,21], we realize that $S^{(1)}, S^{(2)}, S^{(3)}$ depend only on the terms with $k^{\mu}q^{\nu}$, and the other terms are canceled out. As a result, $S^{(1)}$ only receives contributions from 1.a and 1.b diagrams, $S^{(2)}$ receives contributions from 2.a diagram and $S^{(3)}$ receives contributions from 3.a diagram in Figure 1. $\tilde{S}$ is proportional to $\varepsilon^{\mu\nu\alpha\beta} k_{\alpha} q_{\beta}$ and only occurs when there is a loop of fermions, so $\tilde{S}$ only has a contribution from the first row of Figure 1. It should be emphasized that the analytical results of the amplitudes given in figure 1 do not depend on the choice of the value of ξ. The difference when choosing the t'Hooft- Feynman gauge ($\xi = 1$) [19] compared to choosing the unitary gauge ($\xi = \infty$) [20] is the participation of Goldstone bosons. However, the results shown are the same. In this work, we use the unitary gauge and calculation method as mentioned in Ref. [20], the specific results of $S^{(i)}$ and $\tilde{S}$ are given as:

$$\begin{aligned} S^{(1)} &= \frac{\alpha}{2\pi v s_{\mathrm{W}} t_{12}} \sum_{b,c} \sum_{i=1,2} \left|V_{CKM}\right|_{bc}^{2} \int_0^1 dx \int_0^1 dy \left[Q_u x + Q_d (1-x)\right] \\ &\times \frac{(\delta_{1i} t_{12}^2 + \delta_{2i}) m_{d_c}^2 (1-x)(1-2xy) + m_{u_b}^2 x(2xy - 2y + 1)}{m_{\mathrm{W}}^2 x(x-1) + m_{u_b}^2 x + m_{d_c}^2 (1-x) + (m_{\mathrm{W}}^2 - m_{H^{\pm}}^2) xy(1-x)} \end{aligned} \tag{29}$$

$$\begin{aligned} S^{(2)} &= \frac{\alpha v}{2\pi s_{\mathrm{W}}} \sum_{a} t_{12} \sum_{i=1,2} g(h_i^0 H_1^+ H_1^-)(\frac{v}{m_{u_a}} + R_{i1}) \int_0^1 dx \int_0^1 dy \\ &\times \frac{x^2 y(1-x)}{m_{\mathrm{W}}^2 x(x-1) + m_{H^{\pm}}^2 x + m_{h_i^0}^2 (1-x) + (m_{\mathrm{W}}^2 - m_{H^{\pm}}^2) xy(1-x)} \end{aligned} \tag{30}$$

$$\begin{aligned} S^{(3)} &= \frac{\alpha}{2\pi v s_{\mathrm{W}}} \sum_{a} t_{12} \sum_{i=1,2} R_{i1} (\frac{v}{m_{u_a}} + R_{i1}) \int_0^1 dx \int_0^1 dy x^2 \\ &\times \frac{2m_{\mathrm{W}}^2 + (m_{H^{\pm}}^2 + m_{\mathrm{W}}^2 - m_{h_i^0}^2) y(x-1)}{m_{\mathrm{W}}^2 x^2 + m_{h_i^0}^2 (1-x) + (m_{\mathrm{W}}^2 - m_{H^{\pm}}^2) xy(1-x)} \end{aligned} \tag{31}$$

$$\begin{aligned} \tilde{S}^{(1)} &= \frac{\alpha}{2\pi v s_{\mathrm{W}} t_{12}} \sum_{b,c} \sum_{i=1,2} \left|V_{CKM}\right|_{bc}^{2} \int_0^1 dx \int_0^1 dy \left[Q_u x + Q_d (1-x)\right] \\ &\times \frac{(\delta_{1i} t_{12}^2 + \delta_{2i}) m_{d_c}^2 (1-x) + m_{u_b}^2 x}{m_{\mathrm{W}}^2 x(x-1) + m_{u_b}^2 x + m_{d_c}^2 (1-x) + (m_{\mathrm{W}}^2 - m_{H^{\pm}}^2) xy(1-x)} \end{aligned} \tag{32}$$

In the above formulas, we use the notation a, b, c is the index running from 1 to 3, R_{i1} is the ratio coefficient between the interaction vertices, $R_{i1} = \dfrac{g(h_i^0 W^+ W^-)}{g(hW^+W^-)_{SM}}$. So, we have: $R_{11} = c_\alpha s_{12} - s_\alpha c_{12}, R_{21} = c_\alpha c_{12} + s_\alpha s_{12}$. These results are also consistent with previous publications as in reference [19,20]. Substituting (29), (30), (31) and (32) for (28), we get the analytic form of the total amplitude of $H_1^\pm \to W^\pm \gamma$.

The analytic results obtained by $S^{(1)}, S^{(2)}, S^{(3)}, \tilde{S}$ are also the source to give the partial width of $H_1^\pm \to W^\pm \gamma$.

$$\Gamma(H^+ \to W^+\gamma) = \frac{m_{H^\pm}^3}{32\pi}\left(1 - \frac{m_W^2}{m_{H^\pm}^2}\right)^3 \left(\left|S\right|^2 + \left|\tilde{S}\right|^2\right) \tag{33}$$

Determining the partial decay width of $H_1^\pm \to W^\pm \gamma$ helps us to study related problems such as: the mass and interaction of the charged Higgs boson, the complement to the mass of the top quark, the complement to the mass of the neutrino , addition to the anomalous magnetic moment ... Such interesting content will certainly be studied in the near future.

5. Conclusions

Within the framework of the 3-3-1 model with neutral leptons, we have obtained the following results: i) Apply the constraint condition to give the simplest Higgs spectrum satisfying a lightest neutral Higgs boson that is identified with the Higgs boson in the standard model and show that there is only one charged Higgs boson and two neutral Higgs bosons which participate in the decay process $H_1^\pm \to W^\pm \gamma$, ii) All couplings and Feynman diagrams of $H_1^\pm \to W^\pm \gamma$ decay are given, iii) The full analytic form of the amplitude and partial width of $H_1^\pm \to W^\pm \gamma$ decay are also given.

These results are important for the study of the charged Higgs boson and provide information for the verification of relevant physical phenomena in the current experimental field.

Declaration of Competing Interest

The authors declare no competing interests.

Author contributions

Writing original draft: Trung-Hieu Tran; find the mass spectrum of the Higgs bosons: Minh-Vuong Nguyen; analytic results of couplings and amplitude: Thanh-Hung Ha; all authors have read and agreed to the published version of the manuscript.

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