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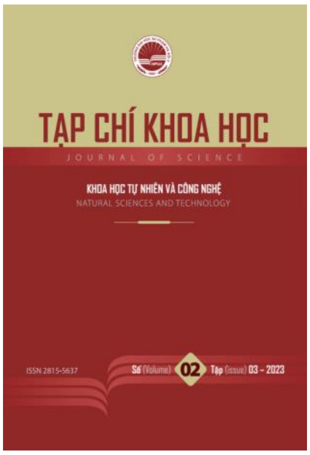

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An analytical approach to finite time H_∞ event-triggered state feedback control of fractional order systems with delay

Truong-Thanh Nguyen*

School of Applied Mathematics and Informatics, Hanoi University of Science and Technology, Ha Noi, Viet Nam

Abstract

This paper investigates finite time H_∞ event-triggered state feedback control problem of fractional-order systems with delay. Based on Laplace trasform and "inf-sup" norm, a delay-dependent sufficient condition for designing H_∞ event-triggered control is established in terms of the Mittag-Leffler function and Linear matrix inequalities. A numerical example is given to show the effectiveness of the obtained result.

Keywords: Fractional order systems, laplace transform, lyapunov function, linear matrix inequalities, time delay

1. Introduction

Nowadays, fractional calculus for delay systems is one of the hot topics in the qualitative theory of dynamical systems (see [1, 2]).

There are some main methods used to study stability analysis of fractional order systems with delay such as Lyapunov functionls [3], Fractional-order Hanalay inequality [4], and Gronwall inequality [5]. The Lyapunov function well known method gives a very effective approach to investigate the stability problem of ordinary differential equations. But *it is more difficult to apply the*

* Corresponding author, E-mail: thanh.nguyentruong@hust.edu.vn

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method for delay systems. Gronwall inequality approach or fractional-order Hanalay inequality does not give satisfactory solution because its conditions are always time delay - independent and it is difficult in estimating the delay solution $\|x_t\|$. To the best knowledge of authors, for *stabilizability of fractional order systems with delay,* controllers in many existing papers are state feedback ($u(t) = Kx(t)$) or output feedback control [6, 7, 8, 9]. Moreover, there are few results for finite time stability of those systems. This inspires us to propose a new effective approach for the finite time H_∞ event-triggered state feedback control problem of fractional-order systems with delay in this paper.

The present paper contributes as the following:

+ A novel approach based on the fractional techniques and using event-triggered state feedback controller are proposed for solving the problem of finite time H_∞ control of fractional order systems with delay.

+ A new dependent time delay sufficient condition for the problem of finite time H_∞ event-triggered state feedback control is derived. And the condition is provided into solving LMI, in which the event-triggered state feedback controllers can be effectively designed.

The layout of this article is organized: section 2, we provide some preliminaries on fractional derivatives, finite-time stability problem and some auxiliary lemmas needed in next section; section 3, a sufficient condition to design finite time H_∞ event-triggered state feedback control for fractional order systems with delay are presented.

Notations: For any matrix $A \in R^{n\times n}$, $A > 0$ or $A < 0$ means that it is positive-definite or negative-definite matrix, respectively; $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximal and the minimal eigenvalues, respectively; The symbol $*$ stands for symmetric block elements in a matrix.

2. Problem statement and preliminaries

Firstly, we give some basic concepts of fractional calculus [1, 2] as follows.

For $\alpha \in (0,1],$ the Riemann-Liouville integral and the Caputo fractional derivative of a function $f(t)$ are defined as

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} f(s)ds,$$

$$D^\alpha f(t) = D_R^\alpha\left[f(t) - f(0)\right],$$

respectively, where $D_R^\alpha f(t) = \frac{d}{dt} I^{1-\alpha} f(t),$ the Gamma function $\Gamma(s) = \int_0^\infty e^{-t} t^{s-1} dt.$

Consider the fractional order control system with uncertainties:

$$\begin{aligned} &D^\alpha x(t) = Ax(t) + Dx(t-h) + W\omega(t) + Bu(t),\\ &z(t) = Cx(t),\\ &x(\theta) = \varphi(\theta), \quad \theta \in [-h, 0], \end{aligned} \tag{2.1}$$

where $\alpha \in (0,1]$, the state vector $x(t)$, the controller $u(t)$, the disturbance $\omega(t)$, the observer $z(t)$, the system matrices A,B,C,D,W are given constant matrices, the constant time delay $h>0$, the initial function $\varphi \in C\left(\left[-h,0\right],R^n\right)$ and $\|\varphi\| = \sup_{s\in[-h,0]} \|\varphi(s)\|$.

Definition 1. ([10]) Given positive scalars c_1, c_2, T. The system (2.1) without controller $u(t)$ is robustly finite-time stable with respect to $\left(c_1, c_2, T\right)$ if for all $t \in [0,T]$, we have $\|\varphi\|^2 < c_1 \Rightarrow \|x(t)\|^2 < c_2$.

In this paper, we use an event-triggered state feedback controller as follows:

$$u(t) = Kx(t_k),\ t \in [t_k, t_{k+1}),$$

where the feedback gain matrix K is determined later and the triggering sequence defined by $t_0 = 0,\ t_{k+1} = \inf\left\{t > t_k : \|x(t) - x(t_k)\| \ge \eta \|x(t)\|\right\}$.

Definition 2. Given positive scalars c_1, c_2, T. The finite-time H_∞ control problem for system (2.1) is solvable if there exist the event-triggered state feedback controller $u(t) = Kx(t_k),\ t \in [t_k, t_{k+1})$, such that following closed loop system:

$$\begin{aligned} &D^\alpha x(t) = Ax(t) + Dx(t-h) + W\omega(t) + BKx(t_k),\ t \in [t_k, t_{k+1}), \\ &x(\theta) = \varphi(\theta),\ \ \theta \in [-h,0], \end{aligned} \tag{2.2}$$

is robustly finite-time stable w.r.t $\left(c_1, c_2, T\right)$ and the $\gamma-$optimal level condition holds $\sup_{\omega} \dfrac{\sup_{t\in[0,T]} I^\alpha \|z(t)\|^2}{\sup_{t\in[0,T]} I^\alpha \|\omega(t)\|^2} \le \gamma,$ where the supremum is taken over zero initial condition and all admissible disturbances $\omega(t)$ satisfying $\|\omega(t)\|^2 \le d,\ \forall t \ge 0$ (2.3)

Remark 1. It is notable that for $\alpha = 1, T = \infty$, the $\gamma-$optimal level condition:

$$\sup_{\omega} \frac{\sup_{t\in[0,T]} I^\alpha \|z(t)\|^2}{\sup_{t\in[0,T]} I^\alpha \|\omega(t)\|^2} \le \gamma \Leftrightarrow \sup_{\omega} \frac{\int_0^\infty \|z(t)\|^2\, dt}{\int_0^\infty \|\omega(t)\|^2\, dt} \le \gamma,$$

which is widely known [11, 12].

Proposition 1. ([13]) Let $V: R^n \to R^n$ be a convex and differentiable function on R^n such that $V(0) = 0.$ If $\alpha \in (0,1],\ x(t) \in R^n$ be a continuous function on $[0,\infty)$, a matrix $P = P^T > 0$, then $D^\alpha[x^T(t)Px(t)] \le 2x^T(t)PD^\alpha x(t),\ \forall t \ge 0.$

Proposition 2. (Schur lemma, [14]) For $X, Y, Z \in R^{n\times n}$, and positive definite matrices

$Y = Y^T$, we have $X + Z^T YZ < 0 \Leftrightarrow \begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0.$

3. Main results

In this section, we will give sufficient conditions for designing the feedback gain matrix K of the event-triggered state feedback controller $u(t) = Kx(t_k),\ t \in [t_k, t_{k+1}),$ for system (2.1). The following notations are defined for simplicity:

The Mittag–Leffler function $E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^n}{\Gamma(\alpha k + \beta)},\ E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^n}{\Gamma(\alpha k + 1)},$

$$a = E_\alpha(hT^\alpha),\ \ \beta_2^* = \sum_{j=0}^{[T/h]} (a-1)^j E_{\alpha,\alpha}(hT^\alpha)\Gamma(\alpha),\ \ \gamma_1 = \frac{\gamma}{\left(2h\beta_2^* \frac{T^\alpha}{\Gamma(\alpha+1)} + 1\right)},$$

$$\beta_1 = \lambda_{\max}(P^{-1}) a \sum_{j=0}^{[T/h]+1} (a-1)^j,\ \ \beta_2 = \gamma_1 \beta_2^* . \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2,\ \ K = YP^{-1}.$$

Theorem 1. For positive scalars $\gamma,\ c_1, c_2, T,$ finite-time H_∞ control problem for the system (2.1) is solvable if there exists a symmetric positive definite matrix P and a free-weight matrix Y such that the following conditions holds:

$$\begin{bmatrix} [BY+AP]+[BY+AP]^T - hP + I & DP & W & 0 & PC^T & \eta P & 0 \\ * & -hP & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma_1 I & 0 & 0 & 0 & 0 \\ * & * & * & I-2P & 0 & 0 & [BY]^T \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{3.1}$$

$$\frac{\beta_1 c_1 + \gamma_1 \beta_2^* \frac{T^\alpha}{\Gamma(\alpha+1)} d}{\lambda_{\min}(P^{-1})} \le c_2. \tag{3.2}$$

The event-triggered state feedback controller $u(t) = YP^{-1}x(t_k),\ t \in [t_k, t_{k+1}).$

Proof. Consider the functional $V(t) = x(t)^T P^{-1} x(t).$ Take the Caputo derivertive of $V(t)$ along the solution of (2.2), we have for $t \in [t_k, t_{k+1}),$

$$\begin{aligned} D^\alpha V(t) &\le 2x(t)^T P^{-1}\left(Ax(t) + Dx(t-h) + W\omega(t) + BKx(t_k)\right) \\ &= 2x(t)^T P^{-1}\left(\left[BK + A\right]x(t) + Dx(t-h) + W\omega(t) + BK\left[x(t_k) - x(t)\right]\right) \end{aligned}$$

$$-hx(t-h)P^{-1}x(t-h)+hV(t-h)$$
$$-hx(t)P^{-1}x(t)+hV(t)+\|Cx(t)\|^2-\gamma_1\|\omega(t)\|^2+\left(-\|Cx(t)\|^2+\gamma_1\|\omega(t)\|^2\right). \tag{3.3}$$

From (3.3) and the following inequalities

$$x(t)^T P^{-1}BK\left[x(t_k)-x(t)\right]\le x(t)^T\left(P^{-1}\right)^2 x(t)+\left[x(t_k)-x(t)\right]^T\left[BK\right]^T BK\left[x(t_k)-x(t)\right],$$
$$0\le\eta^2\|x(t)\|^2-\|x(t_k)-x(t)\|^2,\quad \text{for all } t\in[t_k,t_{k+1}),$$

it follows that:

$$D^\alpha V(t)\le\mu^T\Omega\mu+hV(t)+hV(t-h)-\|Cx(t)\|^2+\gamma\|\omega(t)\|^2$$

where $\mu=[x,x_h,\omega,v_k]^T$,

$$x:=x(t),\ x_h:=x(t-h),\ v_k:=x(t_k)-x(t),\ \omega:=\omega(t),\ \Omega=\left[\Omega_{ij}\right]_{4\times4},$$

$$\Omega_{11}=P^{-1}\left[BK+A\right]+\left[BK+A\right]^T P^{-1}-hP^{-1}+C^TC+\left[P^{-1}\right]^2+\eta^2 I;$$

$$\Omega_{12}=P^{-1}D;\ \Omega_{13}=P^{-1}W;\ \Omega_{14}=0;\ \Omega_{22}=-hP^{-1};\ \Omega_{23}=0;\ \Omega_{24}=0;$$

$$\Omega_{33}=-\gamma_1 I;\ \Omega_{44}=\left[BK\right]^T BK-I;$$

Noting that $K=YP^{-1}$ and

$$\Omega<0\Leftrightarrow diag(P,P,I,P)\times\Omega\times diag(P,P,I,P)=\bar{\Omega}:=\left[\bar{\Omega}_{ij}\right]_{4\times4}<0,$$

where $\bar{\Omega}_{11}=\left[BY+AP\right]+\left[BY+AP\right]^T-hP+PC^TCP+I+\eta^2P^2;$

$$\bar{\Omega}_{12}=DP;\ \bar{\Omega}_{13}=W;\ \bar{\Omega}_{14}=0;\ \bar{\Omega}_{22}=-hP;\ \bar{\Omega}_{23}=0;\ \bar{\Omega}_{24}=0;$$

$$\bar{\Omega}_{33}=-\gamma_1 I;\ \bar{\Omega}_{44}=\left[BY\right]^T BY-P^2.$$

Using Schur lemma and $-P^2\le I-2P$, the condition (3.1) leads to $\bar{\Omega}<0$.

Hence $D^\alpha V(t)\le hV(t)+hV(t-h)-\|Cx(t)\|^2+\gamma_1\|\omega(t)\|^2.$ (3.4)

Step 1. *Robustly finite-time stability.*

From $-\|Cx(t)\|^2\le 0$, we have

$$D^\alpha V(t)-hV(t)\le hV(t-h)+\gamma_1\|\omega(t)\|^2.$$

Let $G(t)=D^\alpha V(t)-hV(t).$ Applying the Laplace transform to the both sides of the expression, we have

$$\mathrm{L}\left[G(t)\right](s)=\mathrm{L}\left[D^{\alpha}V(t)\right](s)-h\mathrm{L}\left[V(t)\right](s)$$
$$=s^{\alpha}\mathrm{L}\left[V(t)\right](s)-V(0)s^{\alpha-1}-h\mathrm{L}\left[V(t)\right](s),$$

and hence

$$\mathrm{L}\left[V(t)\right](s)=(s^{\alpha}-h)^{-1}\left(V(0)s^{\alpha-1}+\mathrm{L}\left[G(t)\right](s)\right).$$

Using the inverse Laplace transform to the above identity gives the following:

$$V(t)=V(0)E_{\alpha}(ht^{\alpha})+\int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(h(t-s)^{\alpha})G(s)ds.$$

Thus, we obtain for all $t\in[0,T]$,

$$V(t)=V(0)E_{\alpha}(ht^{\alpha})+\int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(h(t-s)^{\alpha})\left[D^{\alpha}V(s)-hV(s)\right]ds$$
$$\le V(0)E_{\alpha}(ht^{\alpha})+\int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(h(t-s)^{\alpha})\left[hV(s-h)+\gamma_1\|\omega(s)\|^2\right]ds$$
$$=V(0)E_{\alpha}(ht^{\alpha})+\int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(h(t-s)^{\alpha})hV(s-h)ds$$
$$+\gamma_1\int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(h(t-s)^{\alpha})\|\omega(s)\|^2\,ds$$
$$\le V(0)E_{\alpha}(ht^{\alpha})+[E_{\alpha}(ht^{\alpha})-1]\sup_{s\in[-h,t-h]}V(s)\ +\gamma_1 E_{\alpha,\alpha}(ht^{\alpha})\Gamma(\alpha)\ I^{\alpha}\|\omega(t)\|^2$$
$$\le V(0)E_{\alpha}(hT^{\alpha})+[E_{\alpha}(hT^{\alpha})-1]\sup_{s\in[-h,t-h]}V(s)$$
$$+\gamma_1 E_{\alpha,\alpha}(hT^{\alpha})\Gamma(\alpha)\sup_{s\in[0,T]}I^{\alpha}\|\omega(s)\|^2.$$

Since the function $H(t):=\sup_{s\in[-h,t]}V(s)$ is non-decreasing with respect to *t*, letting $a=E_{\alpha}(hT^{\alpha})$, we obtain that:

$$H(t)\le aH(0)+(a-1)H(t-h)+\gamma_1 E_{\alpha,\alpha}(hT^{\alpha})\Gamma(\alpha)\sup_{s\in[0,T]}I^{\alpha}\|\omega(s)\|^2,\ t\in[0,T].$$

By induction and the inequalites $E_{\alpha}(hT^{\alpha})\ge 1$, we have

$$H(0)\le\lambda_{\max}(P^{-1})\|\varphi\|^2\le\beta_1\|\varphi\|^2+\beta_2,$$
$$H(t)\le\beta_1\|\varphi\|^2+\beta_2,\ \forall t\in[0,T],$$

then

$$V(t) \le \sup_{s\in[-h,t]} V(s) \le \beta_1 \|\varphi\|^2 + \beta_2, \ \forall t \in [-h,T]. \tag{3.5}$$

where $\beta_1 = \lambda_{\max}(P^{-1}) a \sum_{j=0}^{[T/h]+1} (a-1)^j$, $\beta_2^* = \sum_{j=0}^{[T/h]} (a-1)^j E_{\alpha,\alpha}(hT^\alpha)\Gamma(\alpha)$,

$$\beta_2 = \gamma_1 \sum_{j=0}^{[T/h]} (a-1)^j E_{\alpha,\alpha}(hT^\alpha)\Gamma(\alpha) \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2 = \gamma_1 \beta_2^* . \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2 .$$

Besides, since $V(t) \ge \lambda_{\min}(P^{-1}) \|x(t)\|^2$ and the inequalities (2.3) and (3.2) if $\|\varphi\|^2 \le c_1$, the inequality holds:

$$\|x(t)\|^2 \le \frac{V(t)}{\lambda_{\min}(P^{-1})} \le \frac{H(t)}{\lambda_{\min}(P^{-1})} \le \frac{\beta_1 \|\varphi\|^2 + \beta_2}{\lambda_{\min}(P^{-1})}$$

$$\le \frac{\beta_1 c_1 + \gamma_1 \beta_2^* . \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2}{\lambda_{\min}(P^{-1})} \le \frac{\beta_1 c_1 + \gamma_1 \beta_2^* \frac{T^\alpha}{\Gamma(\alpha+1)} d}{\lambda_{\min}(P^{-1})} \le c_2, \ \forall t \in [0,T].$$

Therefore, the closed loop system (2.2) is robustly finite-time stable w.r.t (c_1, c_2, T).

Step 2. *The* $\gamma -$*optimal level condition*

From (3.4), it follows that:

$$\|z(t)\|^2 \le \|Cx(t)\|^2 \le -D^\alpha V(t) + hV(t) + hV(t-h) + \gamma_1 \|\omega(t)\|^2 .$$

Hence and the zero initial condition $\varphi \equiv 0$ and (3.5), we have

$$\begin{aligned}
I^\alpha \|z(t)\|^2 &\le -I^\alpha D^\alpha V(t) + hI^\alpha V(t) + hI^\alpha V(t-h) + \gamma_1 I^\alpha \|\omega(t)\|^2 \\
&= -\left[V(t) - V(0)\right] + hI^\alpha V(t) + hI^\alpha V(t-h) + \gamma_1 I^\alpha \|\omega(t)\|^2 \\
&\le V(0) + 2hI^\alpha [\beta_1 \|\varphi\|^2 + \beta_2] + \gamma_1 I^\alpha \|\omega(t)\|^2 \\
&= V(0) + 2h\left(\beta_1 \|\varphi\|^2 + \beta_2\right) \frac{t^\alpha}{\Gamma(\alpha+1)} + \gamma_1 I^\alpha \|\omega(t)\|^2 \\
&\le 2h\beta_2 \frac{T^\alpha}{\Gamma(\alpha+1)} + \gamma_1 \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2 \\
&= 2h\, \gamma_1 \beta_2^* . \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2 \frac{T^\alpha}{\Gamma(\alpha+1)} + \gamma_1 \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2 \\
&\le \left(2h\gamma_1 \beta_2^* \frac{T^\alpha}{\Gamma(\alpha+1)} + \gamma_1 \right) \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2
\end{aligned}$$

$$= \gamma_1 \left(2h\beta_2^* \frac{T^\alpha}{\Gamma(\alpha+1)} + 1 \right) \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2 = \gamma \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2.$$

Consequently,

$$\sup_{s\in[0,T]} I^\alpha \|z(s)\|^2 \le \gamma \sup_{s\in[0,T]} I^\alpha \|\omega(s)\|^2 \Leftrightarrow \frac{\sup\limits_{s\in[0,T]} I^\alpha \|z(s)\|^2}{\sup\limits_{s\in[0,T]} I^\alpha \|\omega(s)\|^2} \le \gamma.$$

This completes the proof.

Remark 1. In Theorem 1, the scalars c_1, c_2, T, γ, d are given positive. Therefore, to check the conditions of the theorem, we prescribe these parameters firstly. Since the scalars $c_1, c_2,$ are not involved in (3.1) we first find the unknowns of LMI (3.1) by using LMI Tollbox algorithm and then verify the inequality (3.2).

Remark 2. The system (2.1) as $D = 0$ can be simplified to

$$\begin{aligned} &D^\alpha x(t) = Ax(t) + W\omega(t) + Bu(t), \\ &z(t) = Cx(t), \\ &x(0) = x_0. \end{aligned} \tag{3.6}$$

In [15], the authors discuss the problem of finite time H_∞ state feedback control ($u(t) = Kx(t)$) for the system (3.6). Their approach, however, is not suitable for fractional-order delayed systems. Furthermore, it is unable to utilize event-triggered state feedback control to address the H_∞ control problem for system (3.6). It is worth noting that Theorem 1 can be used to obtain a sufficient condition for solving the finite-time H_∞ control problem for the system (3.6). This demonstrates the usefulness of Theorem 1 in the paper.

4. A numerical example

Example 4.1. Consider the system (2.1), where $\alpha = 0.1, h = 0.1, \eta = 0.1, \gamma = 1, d = 1,$

$$A = \begin{bmatrix} -1 & 0.1 \\ 0.1 & -1 \end{bmatrix}, D = \begin{bmatrix} 0.01 & 0. \\ 0 & 0.01 \end{bmatrix}, W = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, C = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}.$$

By using LMI Toolbox in Matlab, the LMI (3.1) is feasible with

$$P = \begin{bmatrix} 0.9963 & 0.0411 \\ 0.0411 & 0.9963 \end{bmatrix}, \ Y = \begin{bmatrix} 0.5481 & -0.3743 \\ -0.3743 & 0.2807 \end{bmatrix}.$$

For $c_1 = 1, c_2 = 3, T = 10,$ we can calculate

$$a = E_\alpha(hT^\alpha) = 1.1521, \ \beta_2^* = \sum_{j=0}^{[T/h]} (a-1)^j E_{\alpha,\alpha}(hT^\alpha)\Gamma(\alpha) = 1.5585,$$

$$\gamma_1 = \frac{\gamma}{\left(2h\beta_2^* \frac{T^\alpha}{\Gamma(\alpha+1)} + 1\right)} = 0.7080, \ \beta_1 = \lambda_{\max}(P^{-1})a \sum_{j=0}^{[T/h]+1} (a-1)^j = 1.4225.$$

And the condition (3.2) satisfies due to

$$\frac{\beta_1 c_1 + \gamma_1 \beta_2^* \frac{T^\alpha}{\Gamma(\alpha+1)} d}{\lambda_{\min}(P^{-1})} = 2.9906 \le c_2 = 3.$$

Hence finite - time H_∞ control problem for the system (2.1) is solvable w.r.t. $(1, 3, 10)$ with the event-triggered state feedback controller:

$$u(t) = Kx(t_k) = \begin{bmatrix} 0.5666 & -0.3990 \\ -0.3880 & 0.2978 \end{bmatrix} x(t_k), \ t \in [t_k, t_{k+1}).$$

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