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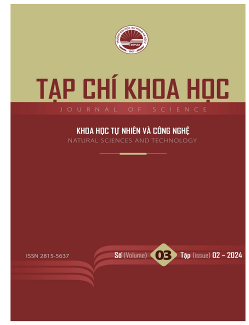

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The decay of SM-like Higgs boson $h_0^1 \to Z\gamma$ in a G221 model

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Abstract

The decay of SM-like Higgs boson $h_0^1 \to Z\gamma$ in the G221 model which is one of all models beyond the SM (BSM) has been investigated. This decay channel has not been observed experimentally. In the framework, we indicate that new particles predicted by this model may significantly affect the previously mentioned decay channel of the SM-like Higgs boson. We will study the effects of the new heavy-charged boson, the charged Higgs boson was not included previously. We are namely, contributing from the diagram both Higgs boson and charged boson. The calculated results are expressed in terms of Passarino-Veltman functions, the numerical evaluation of which can be obtained using LoopTools. Our numerical investigation presents some details and properties of this decay. These may prove useful for comparison with future experimental results that could be detected.

Keywords: Higgs boson decay, rare decay of Higgs boson, extensions of Standard Model, Higgs sector, the G221 model

1. Introduction

Nowadays, the LHC serves as an effective tool for investigating new physics. After the identification of the Standard Model-like (SM-like) Higgs boson particle at LHC in 2012 [1], [2], the Standard Model (SM) of particle physics has once more been corroborated, although there are still problems that the SM cannot explain. Consequently, the SM must be extended. One interesting extension of the beyond the SM models is the models based on the $SU(2)_1 \times SU(2)_2 \times U(1)_Y$ gauge group [3]–[5] (called G221 for short). The experimental data of lepton-flavor non-universality (LNU)

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in the framework of the G221 group has been first studied in [4]. Nevertheless, the main focus of the mentioned paper is an exploration of the LNU. The other puzzles of the model such as neutrino masses, and dark matter were solved in [5] by adding one charged singlet Higgs boson δ.

On the other hand, the decays of the SM Higgs boson into $\gamma\gamma$ and $Z\gamma$ are very attractive from both theoretical and experimental points of view because the $h_0^1 \to Z\gamma$ decay has not been observed [6]. The first evidence for the decay process $h_0^1 \to Z\gamma$ has been observed through a combination of the ATLAS [7] and CMS [8] experiments. The evidence for the decay process $h_0^1 \to Z\gamma$ has been established with an observed significance of 3.4σ and the observed result is consistent with the SM prediction within 1.9σ [6]. The partial decay width of the $h_0^1 \to Z\gamma$ was calculated within the SM framework and its supersymmetric extension [9]–[13]. The branching ratio (Br) predicted by the SM is $Br(h_0^1 \to Z\gamma) = 1.54.10^{-3}(\pm 5.7\%)$ consistent with the Higgs boson mass $m_{h_0^1} = 125.09$ GeV [14], [15]. The general analytic formula of the decay $h_0^1 \to Z\gamma$ for the beyond Standard Models (BSMs) has been presented in [16]. This decay has been considered in many BSMs such as in left-right symmetry [17]–[22] and the models based on the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetries [23]–[30]. The aim of this paper is to consider the decay $h_0^1 \to Z\gamma$ in the framework of the G221 model [5].

2. Brief review of the G221 model

The G221 model is founded on the gauge structure of $SU(2)_1 \times SU(2)_2 \times U(1)_Y$ with the generators, gauge couplings and fields as in [4], [5]. The ordinary (like in the SM) fermions transform as a singlet under $SU(2)_1$ as follows

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim \left(3,1,2,\frac{1}{6}\right),\ \ell_L = \begin{pmatrix} \upsilon \\ e \end{pmatrix}_L \sim \left(1,1,2,-\frac{1}{2}\right), \tag{1}$$

$$u_R \sim \left(3,1,1,\frac{2}{3}\right),\ e_R \sim (1,1,1,-1),\ d_R \sim \left(3,1,1,-\frac{1}{3}\right), \tag{2}$$

where the numbers in brackets that are quantum numbers correspond to gauge groups $SU(3)_C$, $SU(2)_1$, $SU(2)_2$, and the hypercharge. The electric charge operator is identified as the following form

$$Q = \left(T_3^1 + T_3^2\right) + Y \tag{3}$$

The new vector-like fermions lie in doublets of $SU(2)_1$ and singlets of $SU(2)_2$ as follows

$$Q_{L,R} = \begin{pmatrix} U \\ D \end{pmatrix} \sim \left(3,2,1,\frac{1}{6}\right);\ L_{L,R} = \begin{pmatrix} N \\ E \end{pmatrix} \sim \left(1,2,1,-\frac{1}{2}\right). \tag{4}$$

Note that there are n_{VL} generations of new fermions and we fix $n_{VL} = 2$ to simplify the numerical illustration below. The scalar sector includes two doublets ϕ and ϕ' and one self-dual bidoublet Φ (i.e., $\Phi = \sigma_2 \Phi^* \sigma_2$ where σ_2 is the Pauli matrix)

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \sim \left(1,1,2,\frac{1}{2}\right), \phi' = \begin{pmatrix} \varphi'^+ \\ \varphi'^0 \end{pmatrix} \sim \left(1,2,1,\frac{1}{2}\right),$$
$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \Phi^0 & \Phi^+ \\ -\Phi^- & \tilde{\Phi}^0 \end{pmatrix} \sim \left(1,2,1,\frac{1}{2}\right), \tag{5}$$

with $\tilde{\Phi}^0 = \left(\Phi^0\right)^*$. To provide masses for fermions and gauge bosons, the scalar fields develop vacuum expectation values (VEVs) as follows

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\phi \end{pmatrix}, \langle\phi'\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{\phi'} \end{pmatrix}, \langle\Phi\rangle = \frac{1}{2} \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix} \tag{6}$$

The G221 model is broken by a two-step spontaneous symmetry breaking (SSB) process following the pattern

$$SU(2)_1 \times SU(2)_2 \times U(1)_Y \xrightarrow{u} SU(2)_L \times U(1)_Y \xrightarrow{v_f, v_{f'}} U(1)_Q \tag{7}$$

From (7), it is reasonable to assume

$$u >> v_f, v_{f'} \tag{8}$$

The Yukawa coupling between chiral fermions and the SM Higgs-like ϕ doublet is given by

$$-\mathcal{L}_\phi = \overline{q}_L y_d \phi d_R + \overline{q}_L y_u \tilde{\phi} u_R + \overline{l}_L y_l \phi e_R + h.c., \tag{9}$$

where $\tilde{\phi} \equiv i\sigma_2 \phi^*$. The vector-like fermions have Dirac mass terms

$$-\mathcal{L}_M = \overline{Q}_L M_Q Q_R + \overline{L}_L M_L L_R + \ h.c,. \tag{10}$$

Other interactions are

$$-\mathcal{L}_\Phi = \overline{Q}_R \lambda_q^\dagger \Phi q_L + \overline{L}_R \lambda_l^\dagger \Phi l_L + h.c, \tag{11}$$

$$-\mathcal{L}_{\Phi'} = \overline{Q}_L \tilde{y}_d \Phi' d_R + \overline{Q}_L \tilde{y}_u \tilde{\Phi}' u_R + \overline{L}_L \tilde{y}_l \Phi' e_R + h.c., \tag{12}$$

where $\lambda_{q,\ell}^\dagger$ and $\tilde{y}_{u,d,\ell}$ are $n_{VL} \times 3$ matrices. Combining the chiral and vector-like fermions as

$$U_{L,R}^I \equiv \left(u_{L,R}^i, U_{L,R}^k\right)^T, \ D_{L,R}^I \equiv \left(u_{L,R}^i, D_{L,R}^k\right)^T, \ \varepsilon_{L,R}^I \equiv \left(e_{L,R}^i, E_{L,R}^k\right)^T, \tag{13}$$

where $i = 1, 2, 3$, $k = 1,\cdots, n_{VL}$ and $I = 1,...,3 + n_{VL}$, ones can write the fermion mass Lagrangian in the form

$$-\mathcal{L}_{mass}^f = \overline{U}_L \mathcal{M}_U U_R + \overline{D}_L \mathcal{M}_D D_R + \overline{\varepsilon}_L \mathcal{M}_\varepsilon \varepsilon_R + h.c., \tag{14}$$

where

$$\mathcal{M}_U = \begin{pmatrix} \frac{1}{\sqrt{2}} y_u v_\phi & \frac{1}{2}\lambda_q u \\ \frac{1}{\sqrt{2}} \tilde{y}_u v_{\phi'} & M_Q \end{pmatrix}, \mathcal{M}_D = \begin{pmatrix} \frac{1}{\sqrt{2}} y_d v_\phi & \frac{1}{2}\lambda_q u \\ \frac{1}{\sqrt{2}} \tilde{y}_d v_{\phi'} & M_Q \end{pmatrix}, \mathcal{M}_\varepsilon = \begin{pmatrix} \frac{1}{\sqrt{2}} y_\ell v_\phi & \frac{1}{2}\lambda_\ell u \\ \frac{1}{\sqrt{2}} \tilde{y}_\ell v_{\phi'} & M_L \end{pmatrix} \quad (15)$$

are $(3+ n_{VL}) \times (3+n_{VL})$ matrices [4].

Investigating the phenomenology of the Higgs boson within the allowed parameter regions defined in [4], which obtained results from a specific assumption about two new lepton flavors and textures of the Yukawa couplings $\lambda_{q,\ell}$, we will show masses and eigenstates of charged leptons in more detailed. Similarly, we can derive results for the quark sector. The mass matrix $\mathcal{M}_\varepsilon$ in (15) is 5 × 5 matrices which are considered in the flavor basis ε of charged leptons. A simple texture of $\lambda_{q,\ell}$ is chosen as in [4]

$$\lambda_{q,\ell} = 2c_{\beta'} \begin{pmatrix} \tilde{M}_{L_1,Q_1} & 0 \\ 0 & \tilde{M}_{L_2,Q_2}\Delta_{\mu,s} \\ 0 & \tilde{M}_{L_2,Q_2}\Delta_{\tau,b} \end{pmatrix}, \quad (16)$$

in which, there were two new parameters, Δ_μ and Δ_τ would be examined as free parameters; while $\tilde{M}_{L_1}$, $\tilde{M}_{L_2}$ are "reduced" masses of new charged leptons, $m_{E_k} \sim u\tilde{M}_{L_k}$ [4],

$$\tilde{M}_L = diag\left(\tilde{M}_{L1}, \tilde{M}_{L2}\right) = \sqrt{\frac{M_L^\dagger M_L}{u^2} + \frac{\lambda_\ell^\dagger \lambda_\ell}{4}} \quad (17)$$

If all of V_L are considered as the pristine Dirac particles, the V_L is exactly the neutrinos mixing matrix. The formula of V_L is expressed in block form, specifically as [4]

$$V_L = \left(\begin{array}{c|c} V_L^{11} = \sqrt{I_3 - \frac{1}{4}\lambda_\ell \tilde{M}_L^{-2} \lambda_\ell^\dagger} & V_L^{12} = -\frac{u}{2} V_L^{11} \lambda_\ell M_L^{-1} \\ \hline V_L^{21} = \frac{1}{2}\tilde{M}_L^{-1}\lambda_\ell^\dagger & V_L^{22} = \frac{1}{u}\tilde{M}_L^{-1} M_L^\dagger \end{array}\right), \quad (18)$$

where analytical expression of V_L^{ij}, with $i, j = 1,2$, respective to λ_ℓ in Eq. (16) are shown in [4]. The Yukawa coupling matrix y_ℓ as well mentioned, there exists a condition that the SM block of the charged leptons must be diagonal after the block-diagonalization. It does not affect the results given in Ref. [4], which are based mainly on the gauge couplings. The fermion masses of the SM are given below

$$m_u = \frac{y_u v_f}{\sqrt{2}},\ m_d = \frac{y_d v_f}{\sqrt{2}},\ m_e = \frac{y_e v_f}{\sqrt{2}} \quad (19)$$

and $M_{L_1} = u\tilde{M}_{L_1} s_{\beta'}$, $M_{L_2} = u\tilde{M}_{L_2}\rho_{\mu\tau}$. For the quark sector, $M_{Q_1} = u\tilde{M}_{Q_1} s_{\beta'}$ and $M_{Q_2} = u\tilde{M}_{Q_2}\rho_{sb}$. The heavy particles' masses are stated

$$m_{E_1} = M_{N_1} + \frac{m_e^2\left(s_\beta - \frac{1}{2}\right)}{M_{N_1} s_\beta^2 t_{\beta'}^2},\ m_{U_1,D_1} = u\tilde{M}_{Q_1} + \frac{m_{u,d}^2\left(s_\beta - \frac{1}{2}\right)}{u\tilde{M}_{Q_1} s_\beta^2 t_{\beta'}^2},$$

$$m_{E_2} = M_{N_2} - \frac{c_{\beta'}^2\left(\Delta_\mu^2 m_\mu^2 + \Delta_\tau^2 m_\tau^2\right)\left(s_\beta - \frac{1}{2}\right)}{M_{N_2} s_\beta^2 \rho_{\mu\tau}^2},$$

$$m_{U_2,D_2} = u\tilde{M}_{Q_2} - \frac{c_{\beta'}^2\left(\Delta_s^2 m_{c,s}^2 + \Delta_b^2 m_{t,b}^2\right)\left(s_\beta - \frac{1}{2}\right)}{u\tilde{M}_{Q_2} s_\beta^2 \rho_{sb}^2}. \quad (20)$$

The G221 group has seven gauge bosons: four charged bosons $W^{\pm}$ and $W'^{\pm}$ and three neutral gauge bosons including Z and Z' and massless photon A [4]. Note that the mixing of gauge bosons W_i^1 and W_i^2 $(i = 1,\ 2,\ 3)$ is due to the bidoublet Φ with kinematic term

$$\mathcal{L}_\Phi = Tr[\left(D_u \langle\Phi\rangle^\dagger D^\mu \langle\Phi\rangle\right), \quad (21)$$

where

$$\left(D_\mu \Phi\right)_\alpha^\beta = \partial_\mu \Phi_\alpha^\beta - \frac{i}{2} g_1 W_{i\mu}^1 \left(\sigma_i\right)_\alpha^\gamma \left(\Phi\right)_\gamma^\beta + \frac{i}{2} g_2 \left(\Phi\right)_\alpha^\gamma W_{i\mu}^2 \left(\sigma_i\right)_\gamma^\beta \quad (22)$$

For the convenience in reading, let us write the useful notations. The ratio between that gauge couplings: $tan\,\beta' = \frac{g_1}{g_2}, g \equiv \sqrt{g_1^2 + g_2^2}$, $tan\theta_W = \frac{g'}{g}$, where θ_W is the Weinberg angle. The ratio of two VEVs: $tan\,\beta = \frac{v_\phi}{v_{\phi'}}, v \equiv \sqrt{v_\phi^2 + v_{\phi'}^2} = 246GeV$. As usual, the charged bosons are defined as follows

$$W_i^{\pm} \equiv \frac{1}{\sqrt{2}}\left(W_1^i \mp W_2^i\right),\ i = 1,2. \quad (23)$$

From these fields, ones build the following combinations:

$$\begin{pmatrix} W_l^+ \\ W_h^+ \end{pmatrix} = \begin{pmatrix} c_{\beta'} & s_{\beta'} \\ s_{\beta'} & -c_{\beta'} \end{pmatrix} \begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix}, \quad (24)$$

where $c_\beta \equiv cos\beta$ and so forth. Then physically charged gauge bosons are determined by [5]

$$\begin{pmatrix} W^{\pm} \\ W'^{\pm} \end{pmatrix} = \begin{pmatrix} c_{\xi W} & s_{\xi W} \\ -s_{\xi W} & c_{\xi W} \end{pmatrix} \begin{pmatrix} W_l^{\pm} \\ W_h^{\pm} \end{pmatrix}, \quad (25)$$

where

$$t_{2\xi W} \equiv tan\left(2\xi W\right) = \frac{2s_{2\beta'}(c_{2\beta'} - c_{2\beta}) \in^2}{4 + (1 - 2c_{2\beta}c_{2\beta'} + c_{4\beta'}) \in^2} = c_W t_{2Z}. \quad (26)$$

Here the following definition $\in = v/u$ is used. The neutral gauge bosons W_1^3, W_2^3 and B provide the massless photon and two massive gauge bosons as follows [5]

$$\begin{pmatrix} W_1 \\ W_2 \\ B \end{pmatrix} = \begin{pmatrix} s_W c_{\beta'} & c_W c_{\beta'} & s_{\beta'} \\ s_W s_{\beta'} & c_W c_{\beta'} & -c_{\beta'} \\ c_W & -s_W & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_Z & -s_Z \\ 0 & s_Z & c_Z \end{pmatrix} \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix}, \quad (27)$$

The exact formulas for gauge bosons are given in [5]. Here we just point out the useful relation for the case $\in << 1$

$$\begin{gathered} M_W^2 \approx \frac{g^2 v^2}{4}, \ M_Z^2 \approx \frac{M_W^2}{c_W^2}, \\ \frac{M_W^2}{M_{W'}^2} \approx \frac{s_{2\beta'}^2 \in^2}{4} \approx c_W^2 \frac{M_Z^2}{M_{Z'}^2} \end{gathered} \quad (28)$$

where W and Z are identified with the SM gauge bosons. For completeness, we present the potential of the model [5]

$$\begin{gathered} V = \mu_\phi^2 \phi^\dagger \phi + \mu_\phi^2 \phi'^\dagger \phi' + \mu_\Phi^2 Tr\left(\Phi^\dagger \Phi\right) + \frac{\lambda_1}{2}\left(\phi^\dagger \phi\right)^2 + \frac{\lambda_2}{2}\left(\phi'^\dagger \phi'\right)^2 + \frac{\lambda_3}{2}[Tr\left(\Phi^\dagger \Phi\right)]^2 \\ + \lambda_4 \left(\phi^\dagger \phi\right)\left(\phi'^\dagger \phi'\right) + Tr\left(\Phi^\dagger \Phi\right)[\lambda_5 \left(\phi^\dagger \phi\right) + \lambda_6 \left(\phi'^\dagger \phi'\right)] - \mu\left(\phi'^\dagger \Phi \phi + h.c.\right) + \Delta V_h, \end{gathered} \quad (29)$$

where ΔV_h written as [5]

$$\Delta V_h = \mu_4^2 \left(\delta^+ \delta^-\right) + \lambda_0' \left(\delta^+ \delta^-\right)^2 + \left(\delta^+ \delta^-\right)\left[\lambda_1' Tr\left(\Phi^\dagger \Phi\right) + \lambda_2' \phi^\dagger \phi + \lambda_3' \phi'^\dagger \phi'\right]. \quad (30)$$

We obtained the neutral scalars below

$$\varphi^0 = \frac{1}{\sqrt{2}}\left(v_\phi + S_\phi + iA_\phi\right), \ \varphi'^0 = \frac{1}{\sqrt{2}}\left(v_{\phi'} + S_{\phi'} + iA_{\phi'}\right), \ \phi^0 = \frac{1}{\sqrt{2}}(u + S_\Phi + iA_\Phi) \quad (31)$$

Now we present the physical scalar states of the model [5]. Three CP-even fields and CP-odd scalar sector are defined as in [5]. There are four charged scalars φ^+, φ'^+, Φ^+ and δ^+. In the case of $\in = v/u << 1$, the new singlet $\delta^\pm$ is a physical state denoted $h_2^\pm = \delta^\pm$. The remaining three fields mix up two Goldstone bosons eaten by the massive W and W' and heavy singly charged scalar $h_1^\pm$. The above mixing is given as follows: The original states $H^+ = \left(\varphi^+, \varphi'^+, \Phi^+\right)^T$ and physical states $H^+ = \left(G_W^+, \ G_{W'}^+, \ h_1^+\right)^T$ are connected by $\phi^+ = C_{h^\pm}^T H^+$ where

$$C_{h^\pm} = \begin{pmatrix} -c_\beta s_\varsigma & s_\beta s_\varsigma & c_\varsigma \\ s_\beta & c_\beta & 0 \\ c_\beta c_\varsigma & -s_\beta c_\varsigma & s_\varsigma \end{pmatrix} \quad (32)$$

Here, the mass of $h_1^\pm$ is equal to that of the massive CP-odd scalar h_a

$$m_{h^\pm}^2 = m_{h_a}^2 = \frac{\mu\left(u^2 + v^2 s_\beta^2 c_\beta^2\right)}{2us_\beta c_\beta} \quad (33)$$

To finish this section, let us summarise the content of the Higgs sector: In the CP-even sector, there exist three massive fields, and among them, the light field h_0^1 is identified with the SM-like boson denoted by h. In the CP-odd sector, ones have two massless Goldstone bosons eaten by two massive neutral gauge bosons Z and Z' and one massive field denoted by h_a. In the charged scalar sector, there exist two massless Goldstone bosons to be eaten to form two massive gauge bosons W and W' and two massive fields, namely $h_i^{\pm}$, $i=1,2$.

3. Analytic formula and amplitude contribute to decay $h_0^1 \to Z\gamma$ in the G221 model

In this part, we present only couplings related to the decays $h_0^1 \to Z\gamma$ in the unitary gauge. The trilinear terms in potential (29) give the couplings of the SM-like Higgs boson couplings with charged Higgs bosons:

$$\lambda_{h_1^0 h_1^{\pm}} = -s_h u\left[\frac{\mu}{u}c_\beta s_\beta c_\varsigma^2 + \left(\lambda_5 c_\beta^2 + \lambda_6 s_\beta^2\right)c_\varsigma^2 + \lambda_3 s_\varsigma^2\right]$$
$$-c_h v\left[\frac{\mu}{v}c_\varsigma s_\varsigma + \left(\lambda_1+\lambda_2\right)s_\beta^2 c_\beta^2 c_\varsigma^2 + \lambda_4 c_\varsigma^2\left(s_\beta^4 + c_\beta^4\right) + \left(\lambda_5 s_\beta^2 + \lambda_6 c_\beta^2\right)s_\varsigma^2\right], \tag{34}$$

$$\lambda_{h_I^0 h_I^{\pm}} = -s_h u\left[\frac{\mu}{u}c_\beta s_\beta c_\varsigma^2 + \left(\lambda_5 c_\beta^2 + \lambda_6 s_\beta^2\right)c_\varsigma^2 + \lambda_3 s_\varsigma^2\right]$$
$$-c_h v\left[\frac{\mu}{v}c_\varsigma s_\varsigma + \left(\lambda_I+\lambda_2\right)s_\beta^2 c_\beta^2 c_\varsigma^2 + \lambda_4 c_\varsigma^2\left(s_\beta^4 + c_\beta^4\right) + \left(\lambda_5 s_\beta^2 + \lambda_6 c_\beta^2\right)s_\varsigma^2\right],$$
$$\lambda_{h_I^0 h_2^{\pm}} = -s_h u\lambda_I' - c_h v\left(\lambda_3' c_\beta^2 + \lambda_2' s_\beta^2\right).$$

The Yukawa interactions (9), (11) and (12) give the couplings of the SM-like Higgs boson with SM fermions presented in Table 1. Here we separate each $h\bar{f}f$ into: $-i\left(Y_{h\bar{f}f_L}P_L + Y_{h\bar{f}f_R}P_R\right)$. For SM lepton in this case, we always have $Y_{\bar{f}f_L} = Y_{\bar{f}f_R}$

Table 1. Couplings of the SM-like Higgs boson couplings with fermions.

Coupling	Vertex
$h_1^0\overline{f}_i f_i$	$-c_h m_{e_i}/v$
$h_0^1\overline{F}_1 F_1$	$-s_h c_{\beta'}^2\tilde{M}_{L_1,Q_1}$
$h_0^1\overline{E}_2 E_2$	$-s_h c_{\beta'}^2\left(\Delta_\mu^2+\Delta_\tau^2\right)\tilde{M}_{L_2}$
$h_0^1\overline{U}_2 U_2$, $h_0^1\overline{D}_2 D_2$	$-s_h c_{\beta'}^2\left(\Delta_s^2+\Delta_b^2\right)\tilde{M}_{Q_2} - \frac{c_h c_{\beta'}^2\left(\Delta_s^2 m_{c,s}^2+\Delta_b^2 m_{t,b}^2\right)\times\in}{s_\beta v^2\tilde{M}_{Q_2}\rho_{sb}^2}$
$h_0^1\overline{q}_2 Q_2$, $h_0^1\overline{q}_3 Q_2$	$-\Delta_{s,b}c_{\beta'}\left[s_h\tilde{M}_{Q_2}\rho_{sb}P_L + \frac{c_h m_{q_{2,3}}}{v\rho_{sb}}\left(P_R + \frac{m_{q_{2,3}}}{v\tilde{M}_{Q_2}s_\beta}\in P_L\right)\right]$

The couplings of Higgs and gauge bosons are included in the covariant kinetic terms of the Higgs boson:

$$\mathcal{L}_{kinH} = \left(D_\mu\phi\right)^\dagger \left(D^\mu\phi\right) + \left(D_\mu\phi'\right)^\dagger \left(D^\mu\phi'\right) + Tr\left[\left(D_\mu\Phi\right)^\dagger \left(D^\mu\Phi\right)\right]$$

$$\to g_{hvv} g_{\mu\nu} h v^{-Q_\mu} v^{Q_\nu},$$

$$-ig^*_{hsv} v^{-Q_\mu}\left(s^{+Q}\partial_\mu h - h\partial_\mu s^{+Q}\right),\ ig_{hsv} v^{Q_\mu}\left(s^{-Q}\partial_\mu h - h\partial_\mu s^{-Q}\right),$$

$$igz_{ss} Z^\mu\left(s^{-Q}\partial_\mu s^{Q} - s^{Q}\partial_\mu s^{-Q}\right),\ igz_{vs} Z^\mu v^{Q_\nu} s^{-Q} g_{\mu\nu},\ ig^*_{Z_{vs}} Z^\mu v^{-Q\nu} s^{Q} g_{\mu\nu},$$

$$ieQA^\mu\left(s^{-Q}\partial_\mu s^{Q} - s^{Q}\partial_\mu s^{-Q}\right), \tag{35}$$

where $s = h^\pm_{1,2}$, $h = h^0_1, h^0_2, h^0_3$, $v = W, W'$. Only the related terms contributing to the decay $h \to Z\gamma$ with $h = h^0_1$ are listed. The couplings are presented in Table 2. where $\partial_\mu h \to -ip_{0\mu} h$ and $\partial_\mu s^{\pm Q} \to -ip_{\pm\mu} s^{\pm Q}$, the notations $p^\pm$, p_0 are incoming momenta.

Table 2. Couplings of SM-like Higgs bosons with gauge bosons.

Coupling	Vertex
$h^1_0 W^+_\mu W^-_\mu$	$\frac{g^{\mu\nu} g^2 v}{2} C_h$
$h^1_0 W^+_\mu W'^-_\mu$	$\frac{g^{\mu\nu} g^2 v_{C_h}}{2 s'_\beta c'_\beta}\left(c^2_{\beta'} - c^2_\beta\right)$
$h^1_0 W'^+_\mu W'^-_\mu$	$\frac{g^{\mu\nu} g^2\left[4us_h + c_h v\left(1 - 2c_{2\beta} 2c_{2\beta'} + c^2_{2\beta'}\right)\right]}{8c^2_{\beta'} s^2_{\beta'}}$
$h^1_0 W'^+_\mu h^-_1$	$-\frac{g\left(p_0 - p_+\right)\left(c_\beta c_h c_\varsigma s_\beta - s_h s_\varsigma\right)}{2c_{\beta'} s_{\beta'}}$

The triple couplings of Z to charged Higgs boson are given in Table 3.

Table 3. Third-order couplings of gauge bosons.

Coupling	Vertex
$Z_\mu h^-_1 h^+_1$	$\frac{g}{2c_W}\left(2c^2_W - c^2_\varsigma\right)\left(p_{h^+} - p_{h^-}\right)^\mu$
$Z_\mu W'^-_\nu h^+_1$	$\frac{g^{\mu\nu} g^2\left(s^2_W c_\varsigma s_\beta c_\beta v + c^2_W s_\varsigma u\right)}{2c_W s_{\beta'} c_{\beta'}}$

The couplings of Z and photon A_μ with fermions arising from the covariant kinetic of fermion:

$$\mathcal{L}^f_{kin} = \sum_{a=1}^{2}\left(\overline{\ell_{aL}}\gamma^\mu D_\mu \ell_{aL} + \overline{e_a R}\gamma^\mu\partial_\mu e_a R + \overline{L_{aL,R}}\gamma^\mu D_\mu L_{aL,R}\right)$$

$$+\sum_{a=1}^{2}\left(\overline{Q_{aL,R}}\gamma^\mu D_\mu Q_{aL,R} + \overline{u_{aR}}\gamma^\mu D_\mu u_{aR} + \overline{d_{aR}}\gamma^\mu D_\mu d_{aR} + \overline{q_{aL}}\gamma^\mu D_\mu J_{aL}\right) \tag{36}$$

$$\supset \sum_f\left[\frac{g}{c_W}\overline{f}\gamma^\mu\left(g^f_L P_L + g^f_R P_R\right) f Z_\mu + eQ_f\overline{f}\gamma^\mu f A_\mu\right].$$

where *f* runs over all fermions in the G221 model, Q_f is the electric charge of the f and we used the limited $c_Z \to 1$, with c_Z is cos of the mixing angle $Z_\ell - Z_h$. Values of $g^f_{L,R}$ are shown in [5].

The triple couplings of three gauge bosons arise from the covariant kinetic Lagrangian of the non-Abelian gauge bosons:

$$\mathcal{L}_D^g = -\frac{1}{4}\sum_{a=1}^{8} F^a_{\mu\nu} F^{a\mu\nu} \frac{-b \pm \sqrt{b^2-4ac}}{2a} \tag{37}$$

where

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\sum_{b,c=1}^{8} f^{abc} W^b_\mu W^c_\nu , \tag{38}$$

with f^{abc} *(a,b,c = 1,2,3)* are structure constants of the *SU(2)* group. They are defined as

$$\begin{aligned} \mathcal{L}_D^g \to & -g_{Zvv} Z^\mu(p_0) v^{+Q_v}(p_+) v^{-Q_\lambda}(p_-) \times \Gamma_{\mu\nu\lambda}(p_0, p_+, p_-), \\ & -eQA^\mu(p_0) v^{+Q_v}(p_+) v^{-Q_\lambda}(p_-) \times \Gamma_{\mu\nu\lambda}(p_0, p_+, p_-), \end{aligned} \tag{39}$$

where $\Gamma_{\mu\nu\lambda}(p_0, p_+, p_-)^0 = g_{\mu\nu}(p_0 - p_+)_\lambda + g_{\nu\lambda}(p_+ - p_-)_\mu + g_{\lambda\mu}(p_- - p_0)_\lambda$, and $v = W, W'$. The quartic couplings of the *Z* boson and the photon with charged gauge bosons are given in Table 4. With the above couplings the Feynman diagrams at the one-loop level of the model under consideration in the unitary gauge are depicted in Figure 1. In this gauge, we only focus on the physical states in the loops related in the decay $h_1^0 \to Z\gamma$.

Table 4. Triple gauge couplings related to the decay $h_1^0 \to Z\gamma$.

Coupling	Vertex
$Z^\mu W^+_\nu W^-_\alpha$	$-igc_W\Gamma(\mu\nu\alpha)(p_0, p_+, p_-)$
$Z^\mu W'^+_\nu W'^-_\alpha$	$-igc_W\Gamma(\mu\nu\alpha)(p_0, p_+, p_-)$
$Z^\mu W^+_\nu W'^-_\alpha$	$-igc_W\Gamma(\mu\nu\alpha)(p_0, p_+, p_-)$
$A^\mu W^+_\nu W^-_\alpha$	$-igs_W\Gamma(\mu\nu\alpha)(p_0, p_+, p_-)$
$A^\mu W'^+_\nu W'^-_\alpha$	$-igs_W\Gamma(\mu\nu\alpha)(p_0, p_+, p_-)$

In some previous studies of the rate decay of the Higgs boson, the contributions of the diagram containing Vss, sVV and sss (diagrams (b), (c) and (d) in Figure 1.) have been ignored. Because it is considered to have a small contribution to the current energy scale [31], [32]. Several studies in recent times [16], [25] have considered these contributions. Research results show that those contributions are in the same order as other contributions. Therefore, the study of another class model is necessary, namely, we focus on studying the decay $h_1^0 \to Z\gamma$ in the G221 model in this research.

In the unitary gauge, there are five main diagrams (Figure 1.) that which contribute to the decay amplitude of the SM-like Higgs boson $h_1^0 \to Z\gamma$. Where, the contribution of diagrams (a), (b) and (e) are the same as in SM. Other contributions may bring new physics when we investigate the SM-like

Higgs boson decay in the G221 model. Contributing from fermions are given in diagram (a) in Figure 1, including only pure fermions in a diagram (a). The analytic formulas of the fermions are given as follows [16]

$$\begin{aligned} F_{21,f} &= -\frac{eQ_f N_c}{16\pi^2}\Big(4\big(K^{+}_{LL,RR}+K^{+}_{LR,RL}+c.c\big)\big(C_{12}+C_{22}+C_{2}\big) \\ &+2\big(K^{+}_{LL,RR}-K^{+}_{LR,RL}+c.c\big)\big(C_{1}+C_{2}\big)+2\big(K^{+}_{LL,RR}+c.c\big)C_{0}\Big), \\ F_{5,f_{ijj}} &= 0 \end{aligned} \tag{40}$$

where N_c is the color number

$$\begin{aligned} K^{+}_{LL,RR} &= m_f\left(Y_{hfL}g^{*}_{ZfL}+Y_{hfR}g^{*}_{ZfR}\right), \\ K^{+}_{LR,RL} &= m_f\left(Y_{hfL}g^{*}_{ZfR}+Y_{hfR}g^{*}_{ZfL}\right). \end{aligned} \tag{41}$$

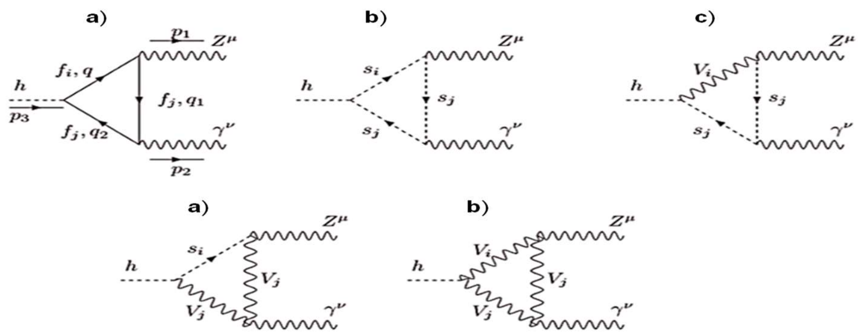

Figure 1. One-loop diagrams contribute to the decay of the SM-like Higgs boson $h_1^0 \to Z\gamma$ in the unitary gauge: **a**) with charged fermions in the loop, **b**) with scalar bosons in the loop, **c**) with a vector boson and two scalar bosons in the loop, **d**) with a scalar boson and two vector bosons in the loop, **e**) with vector bosons in the loop.

The Feynman rules corresponding to the couplings $Y_{hf_{L,R}}$ and $g_{Zf_{L,R}}$ are calculated as rules in [16]. These couplings are shown in Table 2. In the G221 model we have: $Y_{hf_L} = Y_{hf_R}$ and the condition on their value to be real leads lead to: $K^{+}_{LL,RR} = K^{+}_{LR,RL} = m_f Y_{hf_L}(g_L + g_R)$. The result can written as follows:

$$F^{G221}_{21,f} = -\frac{eQ_f N_c K^{f+}_{LL,RR}}{4\pi^2}\left[4\left(C_{12}+C_{22}+C_{2}\right)+C_{0}\right] \tag{42}$$

For pure charge Higgs boson diagram, in the G221 model. There are two diagrams of pure charge Higg boson, namely $h_1^{\pm}$ and $h_2^{\pm}$. The scalar factor $F_{21,h^{\pm}_{1,2}}$ is determined from one-loop contributions. Namely,

$$\begin{aligned} F^{G221}_{21,h_1^{\pm}} &= \frac{\lambda_{h_1^0 h_1^{\pm}}}{16\pi^2}\times\frac{eg\left(2c_W^2 - c_{\varsigma}^2\right)}{2c_W}\left[4\left(C_{12}+C_{22}+C_{2}\right)\right] \\ F^{G221}_{21,h_2^{\pm}} &= 0. \end{aligned} \tag{43}$$

The constants $\lambda_{h_1^0 h_{1,2}^{\pm}}$ arise from the couplings of the triplet of the Higgs boson are given in the Table 1. The G221 model also contributes to the pure diagram of the charged gauge boson. There are two diagrams containing two of the same gauge bosons $W^{\pm}, W'^{\pm}$, and a diagram containing two different gauge bosons $W'^{+}W^{-}$ also has a contribution to the decay $h_1^0 \to Z\gamma$. Namely,

$$F_{21,W}^{G221} = -\frac{2eg^2 c_W c_h m_W}{16\pi^2} \times \left(\frac{2\left(4m_W^2 - m_Z^2\right)}{m_W^2} + \left(8 + \frac{\left(2m_W^2 + m_{h_1^0}^2\right)\left(2m_W^2 - m_Z^2\right)}{m_W^4} \right) \times \left(C_{12} + C_{22} + C_2\right) \right)$$

$$F_{21,W'}^{G221} = -\frac{eg^3 c_W \left[4us_h + c_h v(1 - 2c_{2\beta}c_{2\beta'} + 2_{2\beta'}^2\right]}{16\pi^2 .4c_{\beta'}^2 s_{\beta'}^2} \times \left[\frac{2\left(4m_V^2 - m_Z^2\right)C_0}{m_V^2} + \left(8 + \frac{\left(2m_V^2 + m_{h_1^0}^2\right)\left(2m_V^2 - m_Z^2\right)}{m_V^4} \right)\left(C_{12} + C_{22} + C_2\right) \right] \quad (44)$$

$$F_{21,WW'}^{G221} = -\frac{eg^3 c_W c_h v\left[c_{\beta'}^2 - c_\beta^2\right]}{16\pi^2 .2c_\beta' s_\beta'} \times \left(\frac{2\left(4m_V^2 - m_Z^2\right)C_0}{m_V^2} + \left(8 + \frac{\left(2m_V^2 + m_{h_1^0}^2\right)\left(2m_V^2 - m_Z^2\right)}{m_V^4} \right)\left(C_{12} + C_{22} + C_2\right) \right).$$

Contributing from the diagram of both Higgs boson and charged boson includes 4 diagrams $\{Vss\} = \{W,\, h_1^{\pm}\} + \{W,\, h_2^{\pm}\} + \{W',\, h_1^{\pm}\} + \{W',\, h_2^{\pm}\}$ and 4 diagrams $\{sVV\} = \{h_1^{+}, W^{\pm}\} + \{h_2^{\pm}, W^{\pm}\} + \{h_1^{\pm}, W'^{\pm}\} + \{h_2^{\pm}, W'^{\pm}\}$. However, since the contributions of $g_{ZWs_{ij}} \to 0$, results of another 4 diagram corresponding,

$$F_{21,\{(W' h_1^+ h_1^-),(h_1^+ W'^+ W'^-)\}}^{G221} = -\frac{eg^3}{16\pi^2 c_W} \times \frac{\left(c_\beta s_\beta c_h c_\varsigma - s_h s_\varsigma\right)\left(s_W^2 c_\beta s_\beta c_\varsigma v + c_W^2 s_\varsigma u\right)}{2c_W c_{\beta'}^2 s_{\beta'}^2} \times \left[2\left(1 + \frac{-m_{h_1^{\pm}}^2 + m_h^2}{m'^2_W}\right)\left(C_{12} + C_{22} + C_2\right) + 4C_0 \right] \quad (45)$$

$$F_{21,\{(W' h_1^+ h_1^-),(h_1^+ W'^+ W'^-)\}}^{G221} = 0.$$

From the discussion above, we find the decay width of the SM-like Higgs boson $h_1^0 \to Z\gamma$ in the G221 model. The branching ratio of the decay $h_1^0 \to Z\gamma$ can written as follows

$$Br^{221}\left(h_0^1 \to Z\gamma\right) = \frac{\Gamma^{221}\left(h_0^1 \to Z\gamma\right)}{\Gamma_{h_0^1}^{221}}, \quad (46)$$

where, the partial decay width of the decay is

$$\Gamma^{221}\left(h_0^1 \to Z\gamma\right) = \frac{m_{h_0^1}^3}{32\pi} \times \left(1 - \frac{m_Z^2}{m_{h_0^1}^2}\right)\left(\left|F_{21}^{G221}\right|^2 + \left|F_5^{G221}\right|^2\right) \tag{47}$$

and Γ_h^{221} is the total decay width of the SM-like Higgs boson [33], [34], $F_{21}^{G221} = F_{21,f}^{G221} + F_{21,s}^{G221} + F_{21,V}^{G221} + F_{21,Vs}^{G221}$ have been counted as from Eq. (42) to Eq. (45) and $F_5^{G221} = 0$. The signal strength of the SM Higgs boson decay in the G221 model is predicted by $\mu_{Z\gamma}^{221}$. This quantity is defined in the experiment (LHC) as follows:

$$\mu_{Z\gamma}^{221} \equiv \frac{\sigma^{221}\left(pp \to h_0^1\right)}{\sigma^{SM}\left(pp \to h_0^1\right)} \times \frac{Br^{221}\left(h_0^1 \to Z\gamma\right)}{Br^{SM}\left(h_0^1 \to Z\gamma\right)} \tag{48}$$

where $\sigma^{221}\left(pp \to h_0^1\right)$ and $\sigma^{SM}\left(pp \to h_0^1\right)$ is a cross-section of the SM Higgs boson h_0^1 in the experiment in the G221 model and SM. The signal strength of the decay when $v_{\phi,\phi'}^2 << u^2$ that means $\mathcal{O}\left(\frac{v_{\phi,\phi'}^2}{u^2}\right) \sim 0$ is defined as Eq. (49):

$$\mu_{Z\gamma}^{221} \approx \left|\frac{F_{21}^{G221}}{F_{21}^{SM}}\right|^2 \tag{49}$$

We also stress that for the future Circular Electron Positron Collider (CEPC), $\mu_{Z\gamma}$ can reach 1 ± 0.22 [35]. Additionally, the ATLAS expected significance to the $h_1^0 \to Z\gamma$ channel was looked forward to being.

4. Numerical investigation

In this section, we will apply the allowed regions of the parameter space mentioned above to investigate the decay $h_1^0 \to Z\gamma$. We will fix the numerical parameters based on references [5], [36], and [37]. Relations between parameters in the models are

$$m_{h_1^\pm}^2 = \frac{\mu^2(\mu^2 + v^2 s_\beta^2 c_\beta^2)}{2us_\beta c_\beta},\; m_{Q,L} = u \times diag\left(\tilde{M}_{Q_1,L_1}, \tilde{M}_{Q_2,L_2}\right), \tag{50}$$

where m_{Q_k,L_k} are physical masses of vector-like charged fermions, which will be fixed as $m_{U_{1,2}} = m_{D_{1,2}} = m_{L_{1,2}} = m_F$ in numerical computation. In the limit $\sin\xi$, the mass of the *W'* is

$$m_{W'}^2 = m_{Z'}^2 = \frac{g^2\left[4u^2 + \left(1 - 2c_{2\beta}c_{2\beta'} + c_{2\beta'}^2\right)v^2\right]}{4s_{2\beta'}^2} \tag{51}$$

We will use $m_{h_1^\pm}$ and $m_{W'}$ as input parameters, while μ and u depends on them. For the couplings of $hh_1^\pm$, we fix $s_h=0.1$, $\lambda_1=\lambda_2=\lambda_4=\lambda_5=\lambda_6=1$. We also fix $\beta' \approx \beta$ from the numerical constraint of $\zeta \to 0$.

We note that the current experimental constraint the decay $h_1^0 \to \gamma\gamma$ gives $|s_h| \le 0.1.$ In addition, other parameters are constrained as [37]:

$$m_{Z'} \in [500,1710]GeV,\ g_2 \in [1.2,3.5], \Delta_s \in [-1.16,-0.97], \Delta_b \in [0.003,0.007], \tag{52}$$

$$|\Delta_\mu| \in [0.94,0.99], |\Delta_\tau| \in [0,0.11], t_\beta \in [0.2,0.65], \tag{53}$$

$$\rho_{\mu\tau,sb} \equiv \sqrt{1-c_{\beta'}^2\left(\Delta_{\mu,s}^2+\Delta_{\tau,b}^2\right)} \tag{54}$$

Apart from the free parameters mentioned above, our numerical computation will choose the following default values: $t_b = 0.2,\ \Delta_\mu = 0.99,\ \Delta\tau = 0.11,\ \Delta_s = -1.16,\ \Delta_b = 0.007,\ m_{W'} = 800GeV,$ $m_F = 700GeV$ and $m_{h_1^\pm} = 1.5TeV.$ A note here is that when we choose $s_\xi \to 0$, then all new couplings related to the new charged Higgs boson $h_2^\pm$ are zero. So, we focus only on investigating the $h_1^0 \to Z\gamma$ decay depending on the mass of $h_1^\pm$ and another parameter. From the above parameter space, we hope these values satisfy the current experimental constraints and give a arge $\mu_{Z\gamma}^{221}$.

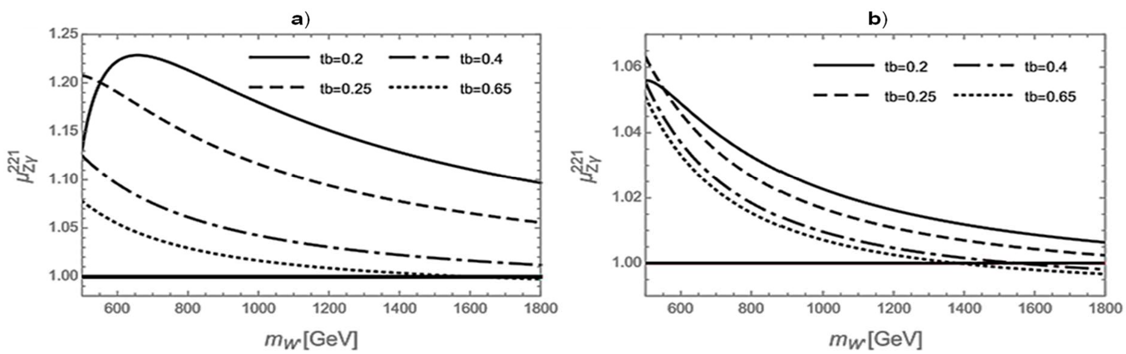

Figure 2. $\mu_{Z\gamma}^{221}$ as function of $m_{W'}$ with **a)** $s_h = 0.1$ and **b)** $s_h = 0.01$.

We first investigate the dependence of $\mu_{Z\gamma}^{221}$ on the heavy gauge boson mass, s_h and t_b as in a Figure 2. In the framework, the couplings of the SM-like Higgs boson to gauge bosons and normal fermions deviate from the SM predictions by a factor denoted as c_h $\left(c_h = \sqrt{1-s_h^2}\right)$ [4]. This factor is constrained to satisfy the global fit of experimental results [38], which gives the constraint [4] $0.995 \le |c_h| \le 1,$ and $s_h \le 0.10.$ We can see that the signal strength depends strongly on the experimental constraint $h_1^0 \to Z\gamma.$ In particular, the largest allowed value s_h predicts large deviations from SM prediction is $\Delta\mu_{Z\gamma}^{221} = \mu_{Z\gamma}^{221} - 11 \approx 0.22 = 22\%$, while smaller $s_h = 0.01$ gives $\Delta\mu_{Z\gamma}^{221} \le 6\%$. We also conclude that large $\mu_{Z\gamma}^{221}$ requires small t_b and $m_{W'}$.

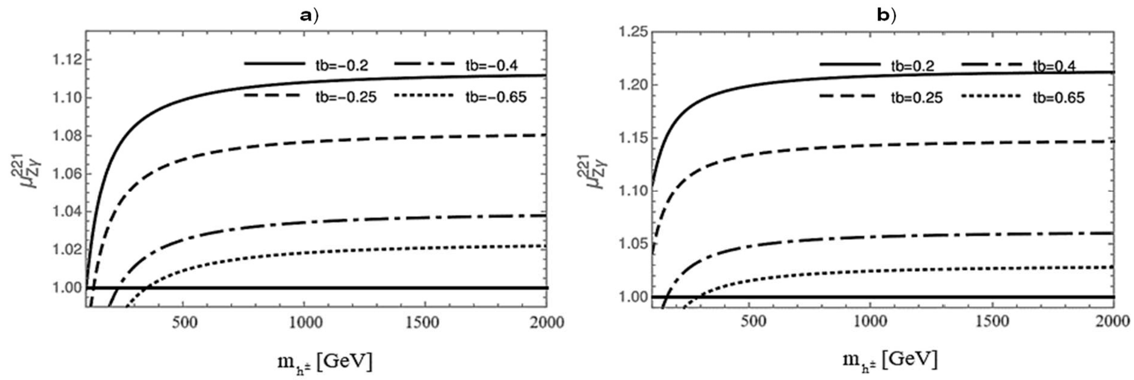

Figure 3. $\mu^{221}_{Z\gamma}$ as a function of $m_{h^\pm}$ with **a**) $s_h = 0.1$ and **b**) $s_h = 0.01$.

The second investigates the dependence of $\mu^{221}_{Z\gamma}$ on the charged Higgs boson mass $m_{h^\pm}, s_h$ and t_b as in a Figure 3. $(m_{h^\pm} = m_{h^\pm_{1,2}})$. With a small value of t_b, s_h and the charged Higgs boson mass $h_1^\pm$ receive the value of a few TeV, which will give large value of $\mu^{221}_{Z\gamma}$. Next, the $\mu^{221}_{Z\gamma}$ will be unchanged when $m_{h^\pm}$ is large enough.

Finally, we investigate the dependence of $\mu^{221}_{Z\gamma}$ on the charged gauge boson mass m_F, s_h and t_b, see Figure 4. It is easy to see that $\mu^{221}_{Z\gamma}$ depends weakly on the charged gauge boson mass m_F, but depends strongly on the s_h.

The future sensitivities are $\mu_{\gamma\gamma} = 1 \pm 0.04$, and $\mu_{Z\gamma} = 1 \pm 0.23$ so that $\left|\delta_{\mu_{\gamma\gamma}}\right| \leq 4\%$, and $\left|\delta_{\mu_{Z\gamma}}\right| \leq 23\%$ [39], respectively, the theoretical constraints on $\mu_{Z\gamma}$ will be more strict. All investigation of this section show that $\mu^{221}_{Z\gamma} > 1$ but still satisfies the limits of the experiment of the decay $h_1^0 \to \gamma\gamma$, which means that in the G221 model, the decay $h_1^0 \to Z\gamma$ can be found experimentally. With the increasing sensitivity of the experiments in the future, we hope that our deviation from SM prediction will be detected.

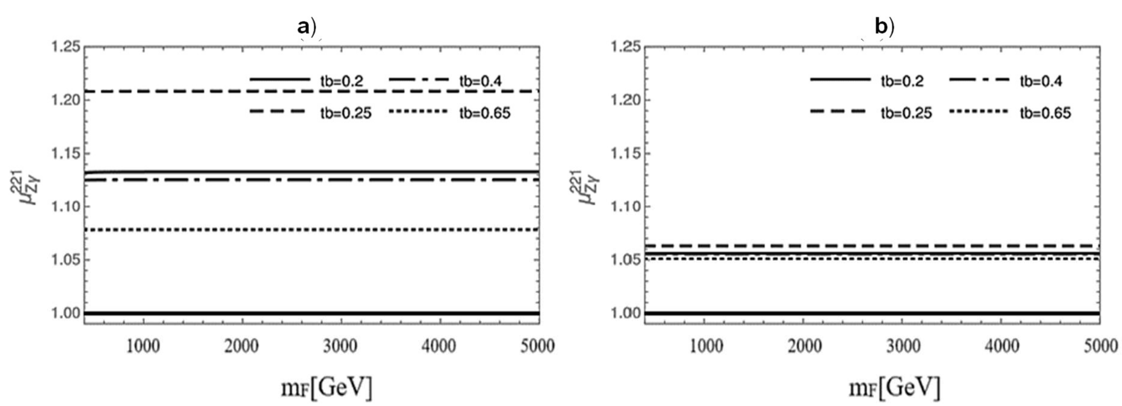

Figure 4. $\mu^{221}_{Z\gamma}$ as a function of m_F with **a**) $s_h = 0.1$ and **b**) $s_h = 0.01$.

5. Conclusion

After studying the decay $h_1^0 \to Z\gamma$ in the G221 model, we have some main conclusions as follows: Firstly, in this work, all of the interdependent terms in the potential were cancelled by using the minimal conditions of the Higgs potential. We used the masses and physical states of all Higgs bosons also their mixing matrices as in [5]. The SM-like Higgs boson and its couplings were readily determined. Secondly, we stress that when we add a scalar singlet $\delta^{\pm}$ in the model so that the Higgs sector also adds more singlet-charged Higgs boson $h_2^{\pm}$. However, it does not contribute to the decay $h_1^0 \to Z\gamma$ when we consider the case of $s_{\xi} \to 0$.

In addition, when we consider $\beta \approx \beta'$, the contribution of the diagram contains the vertex $g_{hWW'} \to 0$. Finally, the signal intensity of the decay $h_1^0 \to Z\gamma$ was researched within a range from 100 GeV to $\mathcal{O}(10)$ TeV of the charged Higgs mass $m_{h_1^{\pm}}$, from 500 GeV to $\mathcal{O}(10)$ TeV of the charged gauge boson mass $m_{W'}$, from 700 GeV to $\mathcal{O}$ (10) TeV of the exotic fermion mass m_F. The largest value of $\mu_{Z\gamma}^{221}$ may reach a value of 1.23 prediction at $m_{W'}$ more than 750 GeV a bit and with small value of t_b. On the other hand, small $m_{h_1}^{\pm}$ also predicts $\mu_{Z\gamma}^{221} > 1$, implying that the signal of this decay channel is easy to observe in future experiments, where the recent upper bound is $\mu_{Z\gamma}^{221} < 6$. It means that the G221 model can be constant news physic. If the decay channel is detected, this model is destined to be verified by experiments in the future.

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