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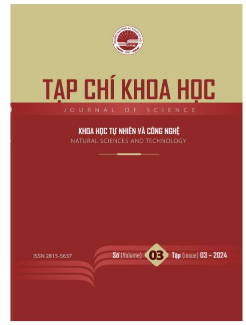

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Algebraic method for image reconstruction in ultrasonic tomography

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Abstract

Ultrasound tomography is crucial due to its capability to deliver detailed, real-time, and non-invasive imaging. This is essential for early diagnosis, treatment planning, and guiding medical procedures. Its affordability, portability, and safety make it a versatile tool in medical and non-medical fields alike, driving ongoing advancements in technology and applications. The Distorted Born Iterative Method (DBIM) is an advanced technique used in ultrasound tomography to iteratively restore images, improving upon the standard Born approximation by addressing some of its limitations. However, the DBIM also has its own set of disadvantages when used for iterative image restoration, resulting in computational complexity, noise sensitivity, convergence issues, etc. In this paper, we introduce a new method for image reconstruction in ultrasound tomography by using the algebraic method. The numerical results indicate that this method has a shorter computational time and achieves high-resolution reconstructions and accurate solutions.

Keywords: Algebraic method, ultrasonic tomography, image reconstruction, incident pressure, scattering pressure

1. Introduction

Ultrasound tomography (UT) [1] is a medical imaging technique that uses sound waves to create three-dimensional (3D) images of internal organs, tissues, and blood flow. It's similar to the more common CT scan (computed tomography) but uses sound waves instead of X-rays. Unlike traditional ultrasound which provides 2D slices, UT builds a 3D image similar to a CT scan. It achieves this by using sound waves instead of X-rays. During a UT exam, a probe emits high-frequency sound waves that travel through the body. The interaction of these waves with tissues – reflection, refraction, scattering and absorption – is measured by multiple detectors positioned around the target area. Powerful computers then process this complex data to create a detailed 3D image. In the case of

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interaction with objects with size in order to wavelength, the ultrasound will be scattered. For detecting small cancer at the beginning of the period, this technique only focuses on the scattering of ultrasound. The advantages of UT are significant. Unlike X-rays, ultrasound is safe for repeated use, making it ideal for monitoring conditions or following up on treatment. Additionally, UT offers superior contrast for soft tissues, potentially improving the detection of tumors or abnormalities compared to traditional ultrasound.

Based on the first-order Born approximation [2], ultrasonic tomography is one of these strategies [3], which is known to be a potentially valuable method of imaging objects with a similar acoustical impedance to that of the surrounding homogeneous medium such as in soft tissue characterization [4] and underwater acoustics [5]. But difficulties arise when it is proposed to obtain quantitative tomograms using acoustical parameters such as the velocity or the attenuation of the wave or tomograms of more highly contrasted media such as in the case of hard tissues [6], industrial process tomography [7], [8], etc. Finding solutions here involves either using iterative schemes [9] and/or performing extensive studies on the limitation of the first-order Born approximation [10]. A high-order Born tomographic method named distorted Born diffraction tomography is also studied in [11], [12].

The theoretical development of the first-order Born tomographic method is asymptotic, as described using Green's function in the case of a homogeneous medium. The unknown object function, which has been extended to the case of weakly contrasted and weakly heterogeneous media, is linearly related to the measured field via a Fourier transform, and the inverse problem is related to the filtered back-projections algorithm [13]. In the case of more highly contrasted media, this strategy was adapted experimentally to take into account physical phenomena such as wave refraction, where the problem can be reduced to the study of a fluid-like cavity buried in an elastic cylinder surrounded by water. Since this iterative experimental method, which is known as compound ultrasonic tomography, gives excellent results, the only limitations here are the heavy data processing required and the complex acoustical signals resulting from the multiple physical effects involved.

Nonlinear inversion methods with higher-order levels of approximation have therefore been investigated, including the distorted Born iterative method [14], [15], which is generally applied to solving electromagnetic [16]–[18] and optical [19] inverse scattering problems. However, very few ultrasonic experimental reconstructions are available in the literature [20]. The ultrasonic distorted Born diffraction tomography approach based on this iterative method makes it possible to obtain quantitative images.

However, UT is still under development. Challenges include complex calculations and the bending of sound waves within the body. Advancements in computing power and new UT designs are paving the way for wider clinical applications. While not yet a replacement for established techniques, UT holds promise for a future where safe, detailed imaging of soft tissues is readily available. There are a couple of methods used for reconstructing ultrasonic tomography images as filtered back projection, distorted Born iterative method (DBIM), etc. where DBIM is one approach for reconstruction in UT. It can handle objects with large variations in sound speed, which is important for biological tissues and achieves high-resolution reconstructions compared to some simpler methods. Due to DBIM is being an iterative method, it can be computationally expensive, especially for complex objects and, in some cases, the iterative process might not converge to an accurate solution. Our algebraic methods are a powerful tool for reconstructing these images from the collected data. Where the object is discretized into a grid of pixels. Ultrasound waves travel through the object along specific paths. In each pixel, the ultrasound may be scattered. The travel time or intensity of the waves is measured for each path. This data is used to create a system of equations that relates the pixel values to the measured data. The algebraic methods then solve this system of equations to estimate the value of each pixel, building the final image. This method can handle complex acquisition geometries where the sound source and receiver may not follow a simple path. Solving large systems of equations can be computationally expensive and gets singulars.

2. Algebraic Reconstruction Techniques

This method involves creating a mathematical model of the object and iteratively refining it based on the acquired ultrasound data. This allows for more complex image reconstruction, especially when dealing with uneven material properties or curved surfaces. For collecting scattered data of UT in the breast, we set up a configuration as Figure 1.

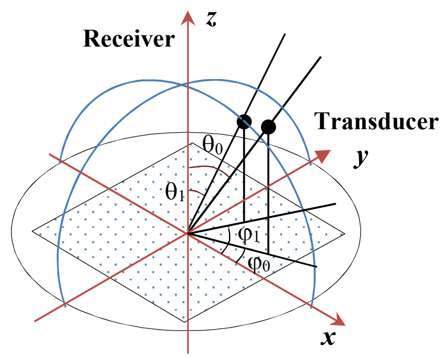

Figure 1. The configuration for ultrasonic computer tomography.

In this configuration, for convenience, we use a transducer and a receiver. They move on an upper-haft sphere with a radius of R. The distance between them is constant when they move on the sphere together. In the sphere coordinate system, the positions of the transducer are determined by the radius R , angles a φ_0 nd θ_0. The positions of receivers are also determined by R, φ_1 and θ_1.

The UT image is considered as a matrix of $N \times N$ pixels. The element of the matrix is defined as the scattering coefficient of soft tissue. These coefficients are equal to zero for objects with sizes that are much greater than the wavelength of ultrasound. They are only different from zero for objects with size in the order of the wavelength of ultrasound. For convenience, the object is defined simply as

$$O(\vec{r}) = \begin{cases} 0 \ \ for \ \vec{r} - \vec{r}_0 > a_0 \\ 1 \ \ for \ \vec{r} - \vec{r}_0 \le a_0 \end{cases} \tag{1}$$

where, $O(\vec{r})$ is the matrix of UT, $\vec{r}$ is the position of a pixel, $\vec{r}_0$ is the position of an object on the plane of the UST and a_0 is the radius (size) of the object.

At the receiver, the pressure of the ultrasound is given

$$p = p^{inc} + \sum_{i=1}^{N \times N} O_i p_i^{scr} \tag{2}$$

where, p^{inc} is the incident sound directly from the transducer, O_i is the scattering coefficient of the *i*-th pixel and p_i^{scr} is the scattered sound by *i*-th pixel of the UT image. The incident sound p^{inc} is estimated as

$$p^{inc}\left(d\right)=p_o.\exp\left(-\mu d\right) \tag{3}$$

where, p_o is the amplitude of sound, μ is the attenuation coefficient and *d* is the distance from the transducer to the receivers. The scattered sound p_i^{scr} is estimated

$$p_i^{scr}=p_0.\exp\left[-\mu\left(r_i^{(0)}+r_i^{(1)}\right)\right] \tag{4}$$

where, $r_i^{(0)}$ is the distance from the transducer to the *i*-th pixel and $r_i^{(1)}$ is the distance from the i-th pixel to the receivers. Let, $b=p-p^{inc}$, $a_1=p_i^{scr}$ and , $x_i=O_i$. Rewriting (2) in the matrix form, we have

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1(NN)} \\ a_{21} & a_{22} & \dots & a_{2(NN)} \\ \dots & \dots & \dots & \dots \\ a_{(NN)1} & a_{(NN)2} & \dots & a_{(NN)(NN)} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_{NN} \end{pmatrix}=\begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_{NN} \end{pmatrix}. \tag{5}$$

The elements of matrix *b* may be obtained from experiments (receivers) and elements of matrix *a* are obtained from the configuration of UT. Dividing matrix *b* by matrix *a*, we may obtain matrix *x*. The matrix of column *x* contains information on the UT image, therefore, it should be rearranged in the form of a square matrix

$$O=\begin{pmatrix} x_{11} & x_{12} & \dots & x_{1N} \\ x_{21} & x_{22} & \dots & x_{2N} \\ \dots & \dots & \dots & \dots \\ x_{N1} & x_{N2} & \dots & x_{NN} \end{pmatrix} \tag{6}$$

3. Computation

To demonstrate our model, two problems are set up and solved as Figure 2. The first is the collection of data from receivers. In the experiment, data is the received pressure of scattering ultrasound on the detectors. In the simulation, we assume that the data is received from (2) via a simple algorithm.

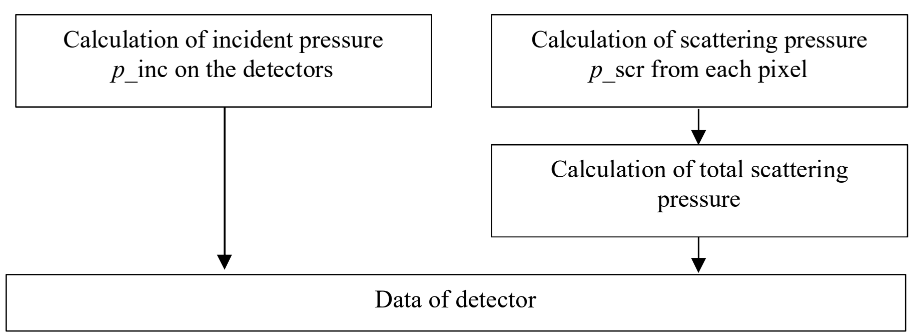

Figure 2. The follow chart of computation.

The second problem is a reconstruction of UT. From (2), the total scattering pressure from pixels at one position of the receiver is given

$$P_{Total}^{(n)}=P_{Data}^{(n)}-P_{inc}^{(n)}=\sum_{i=1}^{N\times N}O_iP_i^{scr(n)} \tag{7}$$

where, $P_{Data}^{(n)}$ is the received data at position *n*, and $P_{inc}^{(n)}$ is the incident wave pressure directly from the transducer. Since the distance between the transducer and the receiver is constant, therefore $P_{inc}^{(n)} = const$ can be calculated via equation (3). The $P_i^{scr(n)}$ is the scattering wave pressure from each pixel i-th at the position of the receiver *n*, they can be calculated via configuration of measurements. From (7), for $n = 1 \ldots N \times N$, we have a system of linear equation.

4. Simulation and discussion

The parameters of configuration are set up for detecting small cancers of the breast. The size of these cancers is in the order of sound wavelength. They usually are in the early stage of cancer and cannot be detected by typical ultrasonic images. The size of UT is $L \times L = 10cm \times 10cm$ and the radius of sphere $R = 5cm$ (size of breast); the matrix of UT is set up as $N \times N = 8 \times 8$ and $N \times N = 13 \times 13$ pixel respectively. The breast may be considered as soft tissue; therefore, the attenuation coefficient of the breast is chosen as $\mu = 1$ in the range from 0.75 (soft tissue) to 1.3 (muscle). The original amplitude of the sound source $p_0 = 100\%$. The displacements of the transducer and receiver are set up by $\Delta\varphi = |\varphi_0 - \varphi_1| = \frac{\pi}{8}$ and $\Delta\theta = |\theta_0 - \theta_1| = \frac{\pi}{8}$.

For *i* = 1 to N

For *j* = 1 to N

$$\varphi_0^{(i)} = (i-1) * 2\pi / N$$

$$\varphi_1^{(i)} = \varphi_0^{(i)} + \Delta\varphi$$

$$\theta_0^{(j)} = (j-1) * \pi / 2N$$

$$\theta_1^{(i)} = \theta_0^{(i)} + \Delta\theta$$

End.

End.

The matrix of the UT is reconstructed as Figure 4 $(N = 8)$ and Figure 6 $(N = 13)$, they are very fit in comparison with the idea target matrixes with the same size in Figure 3 $(N = 8)$ and Figure 5 $(N = 13)$.

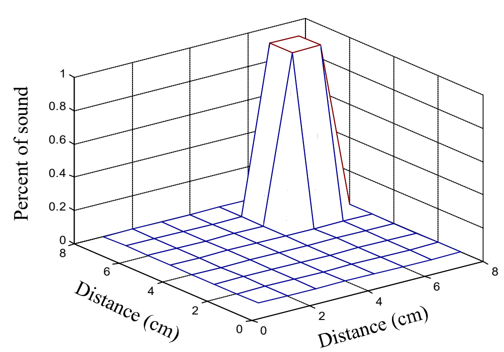

Figure 3. The idea target matrix with $N = 8$.

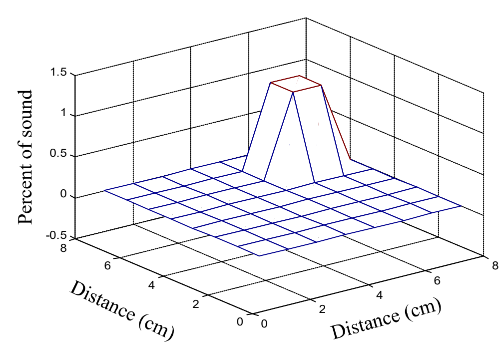

Figure 4. The reconstructed matrix by the algebraic method with $N = 8$.

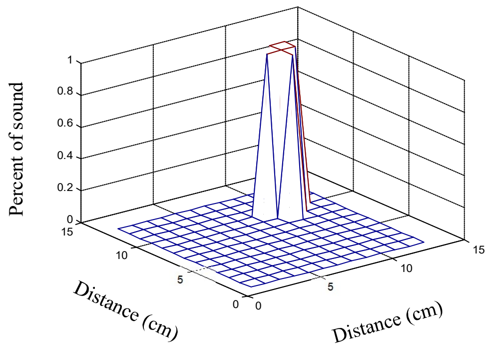

Figure 5. The idea target matrix with $N = 13$.

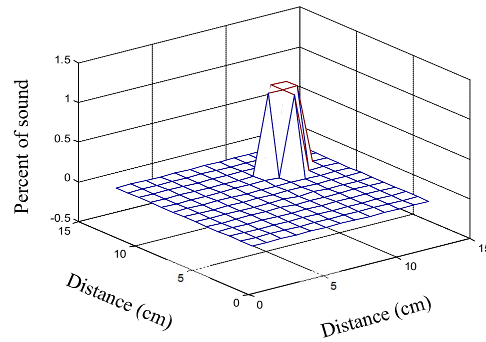

Figure 6. The reconstructed matrix by the algebraic method with $N = 13$.

For greater $N > 13$, our method gets some singulars, which may be due to the very small difference between positions of the transducer and receiver. The determinant of the matrix is close to zero and result may be inaccurate. It will be improved by increasing number of receivers.

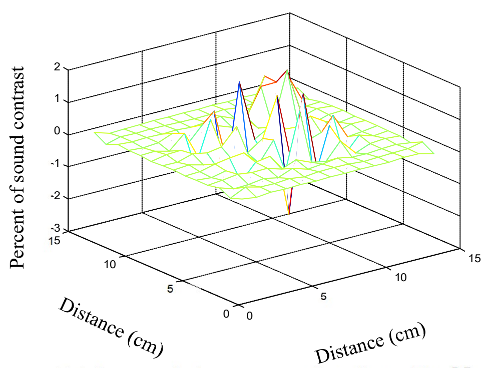

Figure 7. The matrix is close to singular with $N = 14$.

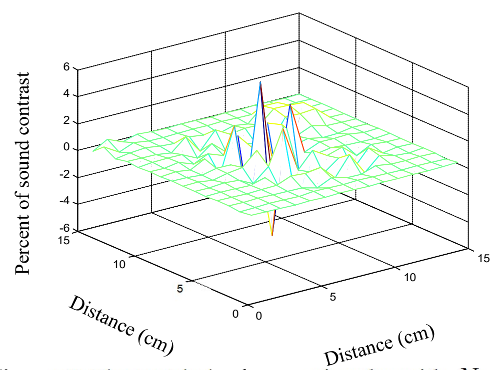

Figure 8. The matrix is close to singular with $N = 15$.

For new idea target matrix

$$O(\vec{r}) = \begin{cases} 0 \ \ for \ \ \vec{r} > a_0 \\ 1 \ \ for \ \ \vec{r} \le a_0 \end{cases} \tag{8}$$

we have reconstructed UT by the DBIM method with the same number of pixels of 8x8 and the same PC. The idea target matrix and the matrix are reconstructed by the DBIM method that are shown in Figures 9 and 10, respectively.

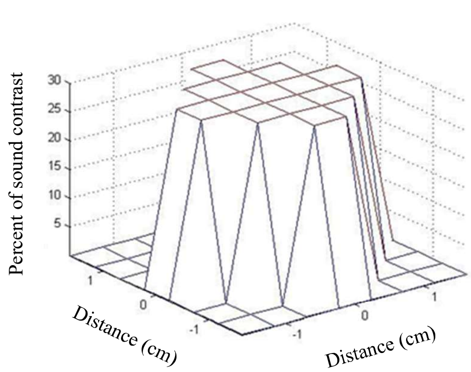

Figure 9. The idea target matrix with $N = 8$.

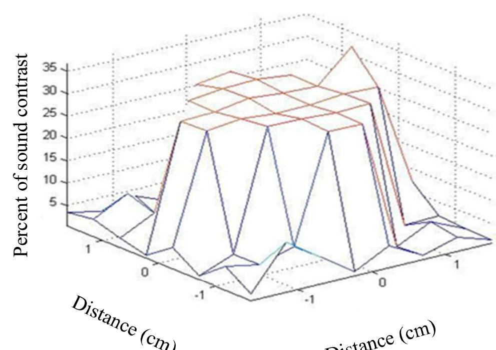

Figure 10. The reconstructed matrix by the DBIM method with $N = 8$.

For the same number of pixels of 8×8 and the same PC, the DBIM takes a computation period of 1.421182 seconds and the Algebra takes a computation period of 0.046105 seconds. Reconstructing UT matrixes by both methods shows that the algebraic method has much shorter computational time and is more accurate.

5. Conclusions

We have set up completely a simply model for reconstructing UT with one transducer and one receiver. The configuration of the model is designed for detecting breast cancer at the early stage. At this stage, the cancers are very small (in the order of sound wavelength) and cannot be detected by typical ultrasonic scanners. In our model, parameters are close to biomedical parameters of organs in the body. By the algebraic method, the duration for computing is very short and the result of the UT image is very accurate in comparison with the DBIM. Since the number of the receivers is few (one), this method may get singulars for higher resolution. These issues will be improved by increasing number of the receivers. Our model has not considered noise yet. In the next research, we will consider noise in the signal processing for application in medicine.

References

[1] B. A. Roberts and A. C. Kak, "Reflection mode diffraction tomography," *Ultrason. Imaging*, vol. 7, no. 4, pp. 300–320, Oct. 1985, doi: 10.1177/016173468500700403.

[2] M. Born *et al*., *Principles of optics: Electromagnetic theory of propagation, interference and diffraction of light*, 7th ed. Cambridge, UK: Cambridge University Press, 1999, doi: 10.1017/cbo9781139644181.

[3] A. C. Kak, "Computerized tomography with X-ray, emission, and ultrasound sources," *Proc. IEEE*, vol. 67, no. 9, pp. 1245–1272, Sept. 1979, doi: 10.1109/proc.1979.11440.

[4] D. Hiller and H. Ermert, "Ultrasound computerized tomography using transmission and reflection mode: Application to medical diagnosis," in *Acoustical imaging*. Boston, MA, USA: Springer, 1982, pp. 553–563, doi: 10.1007/978-1-4613-9780-9_51.

[5] W. Munk and C. Wunsch, "Ocean acoustic tomography: Rays and modes," *Rev. Geophys.*, vol. 21, no. 4, pp. 777-793, May 1983, doi: 10.1029/rg021i004p00777.

[6] P. Lasaygues, J. P. Lefebvre, and S. Mensah, "High resolution low frequency ultrasonic tomography," *Ultrason. Imaging*, vol. 19, no. 4, pp. 278–293, Oct. 1997, doi: 10.1177/016173469701900404.

[7] J. W. Eberhard, "Ultrasonic tomography for nondestructive evaluation," *Annu. Rev. Mater. Sci.*, vol. 12, no. 1, pp. 1–21, Aug. 1982, doi: 10.1146/annurev.ms.12.080182.000245.

[8] W. A. Grandia and C. M. Fortunko, "NDE applications of air-coupled ultrasonic transducers," in *1995 IEEE Ultrason. Symp. Proc. An Int. Symp. Seattle.,* 1995, pp. 697–709, doi: 10.1109/ultsym.1995.495667.

[9] P. Lasaygues, E. Ouedraogo, J. -P. Lefebvre, M. Gindre, M. Talmant, and P. Laugier, "Progress towards in vitro quantitative imaging of human femur using compound quantitative ultrasonic tomography," *Phys. Med. Biol.*, vol. 50, no. 11, pp. 2633–2649, May 2005, doi: 10.1088/0031-9155/50/11/013.

[10] M. Slaney, A. C. Kak, and L. E. Larsen, "Limitations of imaging with first-order diffraction tomography," *IEEE Trans. Microw. Theory Tech.*, vol. 32, no. 8, pp. 860–874, Aug. 1984, doi: 10.1109/tmtt.1984.1132783.

[11] C. Lu, J. Lin, W. Chew, and G. Otto, "Image reconstruction with acoustic measurement using distorted born iteration method," *Ultrason. Imaging*, vol. 18, no. 2, pp. 140–156, Apr. 1996, doi: 10.1177/016173469601800204.

[12] W.C. Chew, *Waves and fields in inhomogeneous media*. New York, USA: Wiley-IEEE Press, 1999, doi: 10.1109/9780470547052.

[13] S. Delamare, "An improvement of back-scattered ultrasonic tomography," *C.R. Acad. Sci., Ser. IIb: Mec., Phys., Chim., Astron.,* vol. 327, no. 5, pp. 475–480, May 1999, doi: 10.1016/s1287-4620(99)80103-3.

[14] O. S. Haddadin and E. S. Ebbini, "Multiple frequency distorted born iterative method for tomographic imaging," in *Acoustical imaging*. Boston, MA, USA: Springer, 1997, pp. 613–619, doi: 10.1007/978-1-4419-8588-0_96.

[15] O. S. Haddadin and E. S. Ebbini, "Imaging strongly scattering media using a multiple frequency distorted Born iterative method," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 45, no. 6, pp. 1485–1496, Nov. 1998, doi: 10.1109/58.738288.

[16] A. G. Tijhuis, K. Belkebir, A. C. S. Litman, and B. P. De Hon, "Multiple-frequency distorted-wave Born approach to 2D inverse profiling," *Inverse Probl.*, vol. 17, no. 6, pp. 1635–1644, Nov. 2001, doi: 10.1088/0266-5611/17/6/307.

[17] A. G. Tijhuis, K. Belkebir, A. C. S. Litman and B. P. de Hon, "Theoretical and computational aspects of 2-D inverse profiling," *IEEE Trans. Geosci. Remote Sens.*, vol. 39, no. 6, pp. 1316–1330, June 2001, doi: 10.1109/36.927455.

[18] Kamal Belkebir and M. Saillard, "Special section: Testing inversion algorithms against experimental data," *Inverse Probl.*, vol. 17, no. 6, pp. 1565–1571, Nov. 2001, doi: 10.1088/0266-5611/17/6/301.

[19] J. C. Ye, K. J. Webb, R. P. Millane, and T. J. Downar, "Modified distorted Born iterative method with an approximate Fréchet derivative for optical diffusion tomography," *J. Opt. Soc. Am. A: Opt. Image Sci. Vis.*, vol. 16, no. 7, pp. 1814–1826, Jul. 1999, doi: 10.1364/josaa.16.001814.

[20] R. Guillermin, P. Lasaygues, J. P. Sessarego, and A. Wirgin, "Imaging an object buried in the sediment bottom of a deep sea by linearized inversion of synthetic and experimental scattered acoustic wavefields," *Inverse Probl.*, vol. 16, no. 6, pp. 1777–1797, Dec. 2000, doi: 10.1088/0266-5611/16/6/312.