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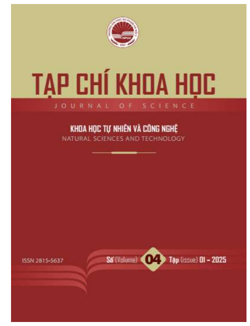

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The standard model-like higgs boson decay $h \to Z\gamma$ in the 3-3-1 simple model

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Abstract

The exact one-loop contributions to the decay amplitudes of the Standard model-like Higgs boson decays $h \to Z\gamma, \gamma\gamma$, as predicted by the simple 3-3-1 model, are presented in terms of using the Passarino-Veltman notations. In the unitary gauge, all triple couplings related to the decay amplitudes are determined and LoopTools package has been applied to the numerical investigation. The result shows that the 3-3-1 simple model predicts the largest value $\Delta_{\mu_{Z\gamma}} = \mu_{Z\gamma} - 1 \leq 15.5\%$, which defines the signal strength of the decay channel $h \to Z\gamma$. This result is still outside the 1σ range of the recent experimental constraint of $\Delta_{\mu_{Z\gamma}} \geq 50\%$, therefore explaining why this signal is still invisible at LHC.

Keywords: Higgs boson decay, loop correction, branching ratio, signal strength, electroweak interaction

1. Introduction

The 3-3-1 simple model (331S) was constructed [1], in the context that the lepton sector does not contain new exotic leptons and only two Higgs triplets are required to generate all fermion and gauge boson masses. Therefore, this model contains fewer Higgs triplets than the original minimal 3-3-1 model [2]. As a result, the physical states of all Higgs boson and their masses can be determined accurately. Consequently, allowing the easy identification of the standard model-like Higgs boson h can be proceeded. This feature is particularly advantageous for studying loop-induced decays of this Higgs boson

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like $h \to Z\gamma, \gamma\gamma$, which are now being searched by experiments and the signal strength of these two decays are $\mu_{Z\gamma} = 2.2 \pm 0.7$ [3], [4] and $\mu_{\gamma\gamma} = 0.99 \pm 0.14$ [5]. Although, the two decay channels were previously discussed [1], our work will employ more general analytic formulas to construct one-loop contributions to both decay amplitudes. Such method allows the addition of new contributions from SVV and VSS diagrams to the $h \to Z\gamma$ amplitude which contain both scalar S and gauge boson propagators in the one-loop Feynman diagrams. The numerical investigation will estimate these two contributions to the $\mu_{Z\gamma}$ predicted by the 331S. Furthermore, we will determine the allowed range of $\mu_{\gamma\gamma}$ and $\mu_{Z\gamma}$ predicted by the 331S, then compare them with the recent experimental results.

2. Brief review of the model

2.1. Particle content and couplings

The 331S model was constructed based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$. The electric charge operator in this model is $Q = T_3 - \sqrt{3}T_8 + X$, where T_3 and T_8 are diagonal generators of the $SU(3)_L$, and X is the charge of the $U(1)_X$. As usual, the covariant derivative relating to this group are:

$$D_\mu = \partial_\mu - ig_s \sum_{i=1}^{8} T_G^i G_\mu^i - ig_3 \sum_{i=1}^{8} T^i W_\mu^i - ig_X T^9 X X_\mu, \tag{1}$$

where

$$P_\mu^{CC} = \sum_{i=1, i\neq 3}^{7} \sum_{i=1}^{8} T^i W_\mu^i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & W^{+\mu} & X_\mu^- \\ W^{-\mu} & 0 & Y_\mu^{--} \\ X_\mu^+ & Y_\mu^{++} & 0 \end{pmatrix} \tag{2}$$

$$P_\mu^{NC} = T^3 W_\mu^3 + T^8 W_\mu^8 + t_X T^9 X X_\mu = diag\big((11)_\mu, (22)_\mu, (33)_\mu\big), \tag{3}$$

and the physical states of the charged gauge bosons

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}, X_\mu^\mp = \frac{W_\mu^4 \mp iW_\mu^5}{\sqrt{2}}, \boldsymbol{Y}_{\boldsymbol{\mu}}^{\mp\mp} = \frac{W_\mu^6 \mp iW_\mu^7}{\sqrt{2}}. \tag{4}$$

The leptons and quarks in the model:

$$\Psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (c_{aR})^c \end{pmatrix} \sim (1,3,0), Q_{\alpha L} = \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ J_{\alpha L} \end{pmatrix} \sim \left(3, 3^*, -{}^1\!/_3\right),$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ J_{3L} \end{pmatrix} \sim \left(3,3, {}^2\!/_3\right);$$

$$u_{aR} \sim \left(3,1, {}^2\!/_3\right), d_{aR} \sim \left(3,1,-{}^1\!/_3\right), J_{\alpha R} \sim \left(3,1,-{}^4\!/_3\right), J_{3R} \sim \left(3,1, {}^5\!/_3\right), \tag{5}$$

where $\alpha = 1,2$, and a=1,2,3 is the family index.

The Higgs sector of the model consists of two Higgs triplets as follows

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^+ \end{pmatrix} \sim (1,3,0), \chi = \begin{pmatrix} \chi_1^- \\ \chi_2^{--} \\ \chi_3^0 \end{pmatrix} \sim (1,3,-1), \tag{6}$$

with the following non-zero vacuum expectation values (VEV) of the neutral components: $\langle \eta_1^0 \rangle = {}^v\!/_{\sqrt{2}}$, $\langle \chi_3^0 \rangle = {}^\omega\!/_{\sqrt{2}}$. These Higgs triplets generate masses of leptons and quarks as follows

$$-\mathcal{L}^Y = h_{33}^J \overline{Q_{3L}} \chi J_{3R} + h_{\alpha\beta}^J \overline{Q_{\alpha L}} \chi^* J_{\beta R} + h_{3a}^u \overline{Q_{3L}} \eta u_{aR} + \frac{h_{\alpha a}^u}{\Lambda} \overline{Q_{\alpha L}} \eta \chi J_{3R}$$

$$+h^d_{\alpha a}\overline{Q_{\alpha L}}\eta^* d_{aR}+\frac{h^d_{\alpha a}}{\Lambda}\overline{Q_{3L}}\eta^*\chi^* d_{aR}$$

$$+h^e_{ab}\overline{(\Psi_{aL})^c}\Psi_{bL}\eta+\frac{s^\nu_{ab}}{\Lambda}\left(\overline{(\Psi_{aL})^c}\eta^*\right)(\Psi_{bL}\eta^*)+H.c. \quad (7)$$

All Higgs masses and Higgs self-couplings will be derived through the following Higgs potential:

$$V_h=\mu_1^2\eta^\dagger\eta+\mu_2^2\chi^\dagger\chi+\lambda_1(\eta^\dagger\eta)^2+\lambda_2(\chi^\dagger\chi)^2+\lambda_3(\eta^\dagger\eta)(\chi^\dagger\chi)+\lambda_4(\eta^\dagger\chi)(\chi^\dagger\eta). \quad (8)$$

Masses and mixing parameters relating to the Gauge bosons are determined from the kinetic part of the Higgs triplets:

$$\mathcal{L}^H_k=(D_\mu\eta)^\dagger(D^\mu\eta)+(D_\mu\chi)^\dagger(D^\mu\chi). \quad (9)$$

The relation between the flavor basis (W^3_μ,W^8_μ,X_μ) and the physical states (A_μ,Z_μ,Z'_μ) are

$$\begin{pmatrix}W^3_\mu\\ W^8_\mu\\ X_\mu\end{pmatrix}=C_Z\begin{pmatrix}A_\mu\\ Z_\mu\\ Z'_\mu\end{pmatrix},C_Z=\begin{pmatrix}s_W & c_W & 0\\ -\sqrt{3}s_W & s_W\sqrt{3}t_W & \sqrt{1-3t_W^2}\\ c_W\sqrt{1-3t_W^2} & -s_W\sqrt{1-3t_W^2} & \sqrt{3}t_W\end{pmatrix},$$

$$m_A^2=0,m_Z^2=m_{Z'}^2\simeq\frac{g^2\omega^2}{3(1-3t_W^2)}, \quad (10)$$

It also shown that the three charged gauge bosons introduced in Eq. (4) are physical, with the masses as follows: $m_W^2={g^2u^2}/{4}$, $m_X^2=g^2{(u^2+\omega^2)}/{4}$, and $m_Y^2={g^2\omega^2}/{4}$. The gauge bosons Z and W are identified with the SM counterparts, leading to the consequence that $u=v\simeq 246$ GeV.

Studying the Higgs potential in Eq. (8), the physical states of the Higgs bosons and mixing parameters are presented in the following formulas:

$$\eta=\begin{pmatrix}\eta_1^0\\ \eta_2^-\\ \eta_3^+\end{pmatrix}=\begin{pmatrix}\frac{u+c_\xi h+s_\xi H+iG_1^0}{\sqrt{2}}\\ G_W^-\\ c_\theta H^+ + s_\theta G_X^+\end{pmatrix},\chi=\begin{pmatrix}\chi_1^-\\ \chi_1^{--}\\ \chi_3^0\end{pmatrix}=\begin{pmatrix}s_\theta H^- + c_\theta G_X^-\\ G_Y^{--}\\ \frac{\omega-s_\xi h+c_\xi H+iG_2^0}{\sqrt{2}}\end{pmatrix}, \quad (11)$$

Here, h,H, and $H^\pm$ are physical states predicted by the 331S model, in which h is identified with the SM-like Higgs boson confirmed experimentally at LHC in 2012. All the remaining massless states including $G_Y^{\pm\pm}$, $G_W^\pm$, $G_Y^\pm$, and $G_{1,2}^0$ are the Goldstone bosons absorbed by the respective gauge bosons appearing in the 331S model. The two mixing parameters ξ and θ are:

$$t_{2\xi}=\frac{\lambda_3 u\omega}{\lambda_2\omega^2-\lambda_1 u^2}\simeq\frac{\lambda_3 u}{\lambda_2\omega},\ \ t_\theta=\frac{u}{\omega}. \quad (12)$$

The SM-like Higgs boson mass is:

$$m_h^2\simeq\frac{4\lambda_1\lambda_2-\lambda_3^2}{2\lambda_2}\simeq 125.09\text{ GeV}. \quad (13)$$

In the numerical investigation presented below, $\lambda_1$is determined as:

$$\lambda_1=\frac{2m_h^2\lambda_2+u^2\lambda_3^2}{4u^2\lambda_2}. \quad (14)$$

The above results lead to the following Feynman rules to calculate the one-loop contribution to the decay amplitude $h\to Z\gamma$ as follows. First, we adopt the notations for vertex couplings introduced [6]. Every triple coupling of a photon always consists of two identical physical particles, as confirmed [7] and [8], see Table 1 for the 331S. Excepting the triple gauge bosons, the results were derived [1] and confirmed by our calculation. New notations can be determined through the following relations:

$$g_{Zf_{aaL}} = \frac{g}{2c_W}(g_V^Z + g_A^Z),\ g_{Zf_{aaR}} = \frac{g}{2c_W}(g_V^Z - g_A^Z).$$

The triple couplings of Z with charged gauge and Higgs bosons and fermions are shown in Table 2. The triple gauge couplings were derived previously [9]–[11]. We also explain that the bold notations were introduced [6]. Below each line with bold notations are the particular values of couplings provided in the 331S framework. The XHH and HXX diagrams relating to the coupling ZXH contribute to decay amplitude $h \to Z\gamma$ that were not mentioned previously. On the other hand, these diagrams do not contribute to the amplitude $h \to \gamma\gamma$. Therefore, the mentioned contributions appearing in the 331S may give large deviation from the SM prediction.

Table 1. Triple couplings of photon in the unitary gauge, $a = 1,2,3$ and $\alpha = 1,2$ in the first line.

Vertex	Coupling	331S
$A^\mu \bar{f}_i f_i$: $f_i = e_a, u_a, d_a, J_\alpha, J_3$	$ieQ_f\gamma_\mu$	$Q_f = -1, \frac{2}{3}, -\frac{1}{3}, -\frac{4}{3}, \frac{5}{3}$
$A^\mu S_i^Q(p_+) S_i^{-Q}(p_-)$: $S_i = H^+$	$ieQ(p_+ - p_-)_\mu$	$Q = 1$
$A^\mu(p_0) V_i^{Q\nu}(p_+) V_i^{-Q\lambda}(p_-)$: $V_i = W^+ X^+ Y^{++}$	$ieQ\Gamma_{\mu\nu\lambda}(p_0, p_+, p_-)$	$Q = 1,1,2$

The triple couplings of h with charged gauge and Higgs bosons and fermions are shown in Table 3 where

$$\lambda_{hh^+H^-} = \frac{1}{2}\lambda_4 s_{2\theta}(s_\xi u - c_\xi \omega) + s_\theta^2(2\omega\lambda_2 s_\xi - uc_\xi\lambda_{34}) + c_\theta^2(\omega\lambda_{34}s_\xi - 2uc_\xi\lambda_1), \quad (15)$$

and $\lambda_{34} = \lambda_3 + \lambda_4$. The notations in this table is the same meaning mentioned for Table 2. We note also that the non-zero coupling hH^-X^+ are necessary for the appearance of two XHH and HXX diagrams we discussed above.

Table 2. Triple couplings of Z in the unitary gauge.

Vertex	Coupling	331S
$Z^\mu \bar{f}_i f_i$	$i\gamma_\mu(g_{Zf_{aa}L}P_L + g_{Zf_{aa}R}P_R)$	$\{g_{Zf_{aa}L}, g_{Zf_{aa}R}\}$
$Z^\mu \bar{t} t$	$\{g_{ZttL}, g_{ZttR}\}$	$\frac{g}{c_W}\left\{\frac{1}{2} - \frac{2}{3}s_W^2, -\frac{2}{3}s_W^2\right\}$
$Z^\mu \bar{J}_\alpha J_\alpha$	$\{g_{ZJ_\alpha J_\alpha L}, g_{ZJ_\alpha J_\alpha R}\}$	$\frac{g}{c_W}\left\{\frac{4}{3}s_W^2, \frac{4}{3}s_W^2\right\}$
$Z^\mu \bar{J}_3 J_3$	$\{g_{ZJ_3J_3L}, g_{ZJ_3J_3R}\}$	$\frac{g}{c_W}\left\{-\frac{5}{3}s_W^2, -\frac{5}{3}s_W^2\right\}$
$Z^\mu S_i^Q(p_+) S_j^{-Q}(p_-)$	$ig_{ZS_{ij}}(p_+ - p_-)_\mu$	$g_{ZS_{ij}}$
$Z^\mu H^+ H^-$	$g_{ZH^+H^-}$	$\frac{g}{c_W}\{s_\theta^2 + 2s_W^2\}$
$Z^\mu V_i^{Q\nu} S_j^{-Q}, Z^\mu V_i^{-Q\nu} S_j^Q$	$ig_{ZV_iS_j}g_{\mu\nu}, ig^*_{ZV_iS_j}g_{\mu\nu}$	$g_{ZV_iS_j}$
$Z^\mu X^{+\nu} X^-$	$g_{ZX^+H^-}$	$\frac{g^2}{2c_W}uc_\theta$
$Z^\mu(p_0) V_i^{Q\nu}(p_+) V_j^{-Q\lambda}(p_-)$	$-ig_{ZV_{ij}}\Gamma_{\mu\nu\lambda}(p_0, p_+, p_-)$	$g_{ZV_{ij}}$
$Z^\mu(p_0) W^{+\nu}(p_+) W^{-\lambda}(p_-)$	$g_{ZW^+W^-}$	gc_W
$Z^\mu(p_0) X^{+\nu}(p_+) X^{-\lambda}(p_-)$	$g_{ZX^+X^-}$	$-\frac{g(1 + 2s_W^2)}{2c_W}$
$Z^\mu(p_0) Y^{++\nu}(p_+) Y^{--\lambda}(p_-)$	$g_{ZY^{++}Y^{--}}$	$\frac{g(1 - 4s_W^2)}{2c_W}$

2.2. Analytical formulas for significant strength for the decay $h \to Z\gamma$

The partial decay widths for $h \to Z\gamma, \gamma\gamma$ are determined generally from the following formulas [12]–[15]

$$\Gamma(h \to Z\gamma) = \frac{m_h^3}{32\pi} \times \left(1 - \frac{m_Z^2}{m_h^2}\right)^3 |F_{21}|^2 + |F_5|^2,$$

$$\Gamma(h \to \gamma\gamma) = \frac{m_h^3}{64\pi} \times \left|F_{\gamma\gamma}\right|^2, \quad (16)$$

where F_{21}, F_5, and $F_{\gamma\gamma}$ are loop contributions. In the framework of the 331S model, the one-loop contributions to the decay $h \to \gamma\gamma$ are listed [1]. In our notations, the analytic formulas are

$$\mathrm{F}_{\gamma\gamma}^{331S} = \sum_f \mathrm{F}_{\gamma\gamma,f}^{331} + \mathrm{F}_{\gamma\gamma,H^+}^{331S} + \sum_{v=W,X,Y} \mathrm{F}_{\gamma\gamma,v}^{331S}, \quad (17)$$

where $f = e_a, u_a, d_a, J_a$, one-loop factors [16]

$$\mathrm{F}_{\gamma\gamma,f}^{331S} = -\frac{e^2 Q_f^2 N_c}{2\pi^2}\left(m_f Y_{h\bar{f}fL}\right)[4x_2 + C_0],$$

$$\mathrm{F}_{\gamma\gamma,H^+}^{331S} = \frac{e^2 \lambda_{hH^+H^-}}{2\pi^2} x_2,$$

$$\mathrm{F}_{\gamma\gamma,v}^{331S} = \frac{e^2 Q_V^2 g_{hvv}}{4\pi^2} \times \left[\left(6 + \frac{m_h^2}{m_V^2}\right) x_2 + 4C_0\right], \quad (18)$$

$x_2 = C_{12} + C_{22} + C_2$, and $C_{0,i,ij} \equiv C_{0,i,ij}(0,0,m_h^2;\ m_x^2, m_x^2, m_x^2)$ are Passarino-Veltman functions [17] with $x = f, s, v$ corresponding to the contribution from fermions, charged Higgs bosons and gauge bosons.

Table 3. Triple couplings of h in the unitary gauge.

Vertex	Coupling	331S
$h\bar{f}_i f_i$	$-i\left(Y_{hf_{ij}L}P_L - Y_{hf_{ij}R}P_R\right)$	$\left\{Y_{hf_{ij}L}, Y_{hf_{ij}R}\right\}$
$h\bar{t}t$	$\{Y_{httL}, Y_{httR}\}$	$\left\{\frac{m_t}{u}c_\xi, \frac{m_t}{u}c_\xi\right\}$
$h\bar{J}_a J_a, a = 1,2,3$	$\left\{Y_{hJ_aJ_aL}, Y_{hJ_aJ_aR}\right\}$	$\left\{-\frac{m_{J_a}}{\omega}s_\xi, -\frac{m_{J_a}}{\omega}s_\xi\right\}$
$hS_i^Q S_j^{-Q}$	$-i\lambda_{hS_{ij}}$	$\lambda_{hS_{ij}}$
hH^+H^-	$\lambda_{hH^+H^-}$	$\lambda_{hH^+H^-}$
$h(p_0)S_i^{-Q}(p_-)V_j^{Q\mu}$	$ig_{hS_iV_j}(p_0 - p_-)_\mu$	$g_{hS_iV_j}$
$hH^-(p_-)X^{+\mu}$	$g_{hH^-X^+}$	$\frac{g}{2}(c_\theta c_\xi + s_\theta s_\xi)$
$hV_i^{-Q\mu}V_j^{-Q\nu}$	$ig_{hV_{ij}}g_{\mu\nu}$	$g_{hV_{ij}}$
$hW^{-\mu}W^{+\nu}$	$g_{hW^+W^-}$	$\frac{g^2 u}{2}c_\xi$
$hX^{-\mu}X^{+\nu}$	$g_{hX^+X^-}$	$\frac{g^2}{2}(uc_\xi - \omega s_\xi)$
$hY^{--\mu}Y^{++\nu}$	$g_{hY^{++}Y^{--}}$	$-\frac{g^2}{2}\omega s_\xi$

All couplings of h appearing in Eq. (18) are shown in Table 3. Based on LoopTools package [18], particular forms given in Eq. (18) will be used for evaluating numerical values related to $\mu_{Z\gamma}$ and $\mu_{\gamma\gamma}$. The one-loop contributions from the SM particles are consistent with the SM results in the limit of $s_\xi = s_\theta = 0$.

Based on the general given in results [6], the analytical formulas of one-loop contributions to decay amplitudes $h \to Z\gamma$ are derived as follows

$F_{21}^{331S} = \sum_f F_{21,f}^{331S} + F_{21,H^+}^{331S} + \sum_{v=W,X,Y} F_{21,v}^{331S} + F_{21,XH^+H^+}^{331S} + F_{21,H^+XX}^{331S}$, and $F_5^{331S} = 0$. (19)

Analytic formulas of one-loop functions are:

$$F_{21,f}^{331S} = -\frac{eQ_f N_f}{16\pi^2}\left(16K_{LL,RR}^{+}x_2 + 4C_0\right),$$

$$F_{21,H^+}^{331S} = \frac{2e\lambda_{hH^+H^-}x_2}{4\pi^2},$$

$$F_{21,v}^{331S} = \frac{2eQ_V g_{hVV} g_{ZVV}}{16\pi^2}\left[\left(8 + \frac{(2m_V^2+m_h^2)(2m_V^2-m_Z^2)}{m_V^4}\right)x_2 + \frac{2(4m_V^2-m_Z^2)C_0}{m_V^2}\right],$$

$$F_{21,XH^+H^+}^{331S} = \frac{2eg_{hH^-X^+}g_{ZX^+H^-}}{16\pi^2}\left[2\left(1 + \frac{-m_{H^+}^2+m_h^2}{m_X^2}\right)x_2 + 4x_0\right],$$

$$F_{21,H^+XX}^{331S} = \frac{2eg_{hH^-X^+}g_{ZX^+H^-}}{16\pi^2}\left[2\left(1 + \frac{-m_{H^+}^2+m_h^2}{m_X^2}\right)x_2 - 4x_3\right], \quad (20)$$

where $K_{LL,RR}^{+} = g_{ZffL}Y_{hffL} + g_{ZffR}Y_{hffR}$, new functions $x_2 = C_{12} + C_{22} + C_{12}$ and $C_{0,ij} = C_{0,ij}\left(m_Z^2, 0, m_h^2; m_x^2, m_x^2, m_x^2\right)$ with $x = f, H, V$ containing one type of virtual particle in the loop. For the two functions $F_{21,XH^+H^+}^{331}$ and F_{21,H^+XX}^{331S} consisting of $x_0 = C_0 + C_1 + C_2$, and $x_3 = C_1 + C_2$, LoopTools notations are

$$C_{0,ij} = C_{0,ij}\left(m_Z^2, 0, m_h^2; m_X^2, m_{H^+}^2, m_{H^+}^2\right) \text{ and } C_{0,ij} = C_{0,ij}\left(m_Z^2, 0, m_h^2; m_{H^+}^2, m_{X^2}^2, m_{X^2}^2\right),$$

respectively. We note that $F_{21,XH^+H^+}^{331S}$ and F_{21,H^+XX}^{331S} were not included in previous discussions [3]. They are also neglected in many beyond the SM cases including the 3-3-1 models [19]–[22]. All Z and h couplings appearing in Eq. (20) were collected in two Tables 2 and 3.

The signal strengths related to the dominant Higgs production channel at LHC, named as the ggH fusion $gg \to h$,corresponding to the two above decay channels are:

$$\mu_{Z\gamma}^{331S} \equiv \frac{c_\xi^2 Br^{33}\quad(h\to Z\gamma)}{Br^{SM}(h\to Z\gamma)}, \mu_{\gamma\gamma}^{331S} \equiv \frac{c_\xi^2 Br^{331S}(h\to\gamma\gamma)}{Br^{SM}(h\to\gamma\gamma)}, \quad (21)$$

where $Br^{SM}(h \to Z\gamma, \gamma\gamma)$ is the decay rates of the SM-like Higgs boson decay $h \to Z\gamma$ in the SM framework. In the next section, the above analytic formulas will be used to investigate numerically to discussion the signals of decays $h \to Z\gamma, \gamma\gamma$ under recent experimental searches.

3. Numerical discussions

In this investigation, numerical values of well-known parameters fixed by experiments will be taken [23]. We will choose the following scanning range for free parameters:

$$\omega \in [3.6\,TeV, 5\,TeV];\ \lambda_2, \lambda_4 \in [0.01, 8];\ |\lambda_3| \le \lambda_2,$$
$$m_{J_1} = m_{J_2}, m_{J_3} \in [0.5\,TeV, 2\,TeV]. \quad (22)$$

Other dependent parameters like ξ, θ , and λ_1 will be determined from Eqs. (12), and (14), respectively. The hWW and hZZ couplings must be consistent with the experiments; hence we will use the constraint $|s_\xi| \le 0.1$.

To express the differences from the SM, we define a quantity $\Delta\mu_{Z\gamma}$ as [16]

$$\Delta\mu_{Z\gamma}^{33} \quad \equiv \left(\mu_{Z\gamma}^{331S} - 1\right) \times 100\%, \quad (23)$$

which is constrained by recent experiments $\Delta\mu_{Z\gamma} = 1.2 \pm 0.7$ [3], [4], implying the 1σ deviation is $50\% \le \Delta\mu_{Z\gamma}^{331S} \le 190\%$. The 1σ constraint from $h \to \gamma\gamma$ decay originating from ggF fusion is defined as $\Delta\mu_{\gamma\gamma}^{331S} \equiv (\mu_{\gamma\gamma}^{331S} - 1) \times 100\%$, leading to the following respective 1σ deviation: $-15\% < \Delta\mu_{\gamma\gamma}^{LR} < 13\%$ corresponding to the experimental constraint $\Delta\mu_{\gamma\gamma} = 0.99 \pm 0.14$. [5]. The numerical results we discuss in the following will always satisfy this constraint. The correlation between two signal strengths are shown in Figure. 1. The ranges predicted by the 331S are $0 < \Delta\mu_{\gamma\gamma}^{331S} \leq 11.3\%$ and $0 \leq \Delta\mu_{Z\gamma}^{331S} \leq 15.5\%$.

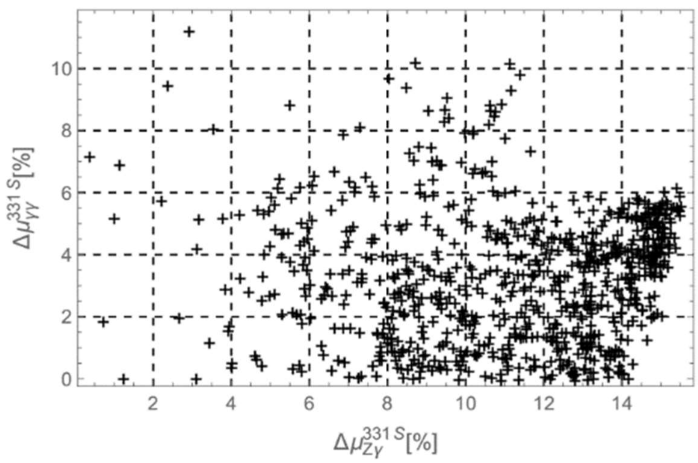

Figure 1. Correlations between $\Delta\mu_{Z\gamma}^{331S}$ and $\Delta\mu_{\gamma\gamma}^{331S}$.

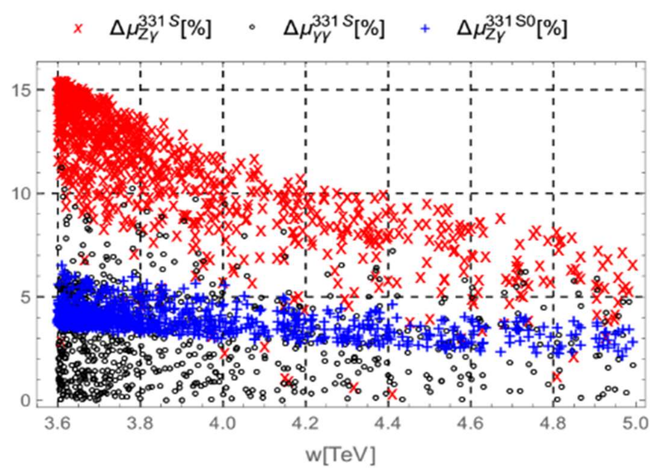

Figure 2. $\Delta\mu_{Z\gamma}^{\mu^{331S}}, \Delta\mu_{\gamma\gamma}^{\mu^{331S}}$, and $\Delta\mu_{Z\gamma}^{\mu^{331S}}$ as functions of ω.

The two signal strengths depend strongly on the $SU(3)_L$ scale ω, as illustrated in Figure 2. We also introduce a new quantity as $\Delta\mu_{Z\gamma 0}^{\mu^{331S}}$ without contributions from $F_{21,XH^+H^+}^{331S}$ and F_{21,H^+XX}^{331S} to estimate qualitatively the contributions of $\Delta\mu_{Z\gamma}^{\mu^{331S}}$. It is clearly shown that $\Delta\mu_{Z\gamma}^{\mu^{331S}} < 6.7$, which is significantly smaller than $\max\left|\Delta\mu_{Z\gamma}^{\mu^{331S}}\right| = 15.5\%$. Therefore, we conclude that the FSV contributions are important in the total amplitude of $h \to Z\gamma$. Finally, the correlation between important parameters and $\Delta\mu_{Z\gamma}$ are shown in Figure 3. We can see that $\Delta\mu_{Z\gamma}^{331S}$ depends strongly on λ_4, s_θ, and s_ξ. Namely, large value of $\Delta\mu_{Z\gamma}^{331S}$ requires large values of $\lambda_4 = 8, s_\theta = 0.068$, and $s_\xi = 0.034$. Therefore, the future results from experiments for the signal strength $\mu_{Z\gamma}$ will provide interesting information for constraining the parameters of the 331S model.

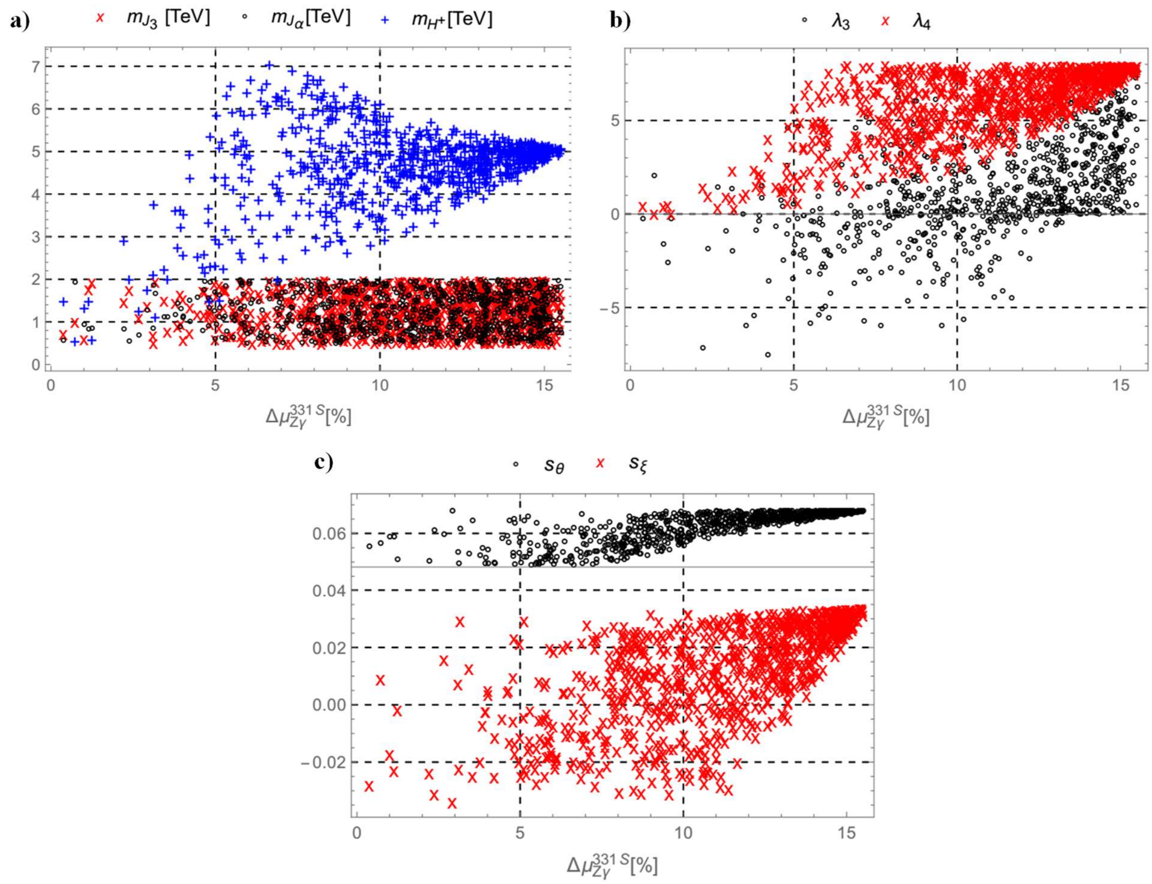

Figure 3. Important parameters in the 331S model as functions of $\Delta\mu_{Z\gamma}^{331S}$. a) Masses of heavy particles predicted by the 331S model as functions of $\Delta\mu_{Z\gamma}^{331S}$. b) Triple Higgs self couplings as functions of $\Delta\mu_{Z\gamma}^{331S}$. c) Sine of mixing parameters ξ and θ as functions of $\Delta\mu_{Z\gamma}^{331S}$.

4. Conclusions

We have established the analytic formulas for one-loop contributions to the decay amplitudes $h \to Z\gamma, \gamma\gamma$ in the 331S framework, employing the notations of PV-functions and LoopTools for numerical investigation. Notably, we showed that the contributions from two diagrams HXX and XHH, which consist of both Higgs and gauge boson propagators in the Feynman diagrams, strongly affect the $\Delta\mu_{Z\gamma}^{33}$, therefore, they must not be neglected in theoretical calculations. The numerical results showed that the maximal values that the 331S model predicts for the two decay channels $h \to Z\gamma, \gamma\gamma$ are $\Delta\mu_{\gamma\gamma} \simeq 11.3\%$ and $\Delta\mu_{Z\gamma} \simeq 15.5\%$. These results are consistent with the SM predictions. In addition, $\Delta\mu_{Z\gamma}$ is smaller than 1σ range given by the recent experimental result. Finally, we emphasize that the future study this decay channel may give interesting constraints on the parameters of the 331S model.

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