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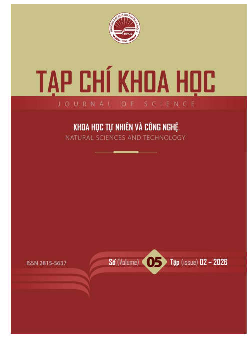

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The state of axion in a DFSZ model

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Abstract

We investigate Peccei–Quinn (PQ) symmetry in the Dine–Fischler–Srednicki–Zhitnitsky (DFSZ) model, which is widely regarded as an elegant solution to the strong CP problem. In this work, the axion state is described in terms of the original pseudo-scalar fields, while the mass spectra of the CP-odd and CP-even Higgs bosons are also derived. Based on the Yukawa Lagrangian and the Higgs potential of the model, we determine the PQ charges of the fields and show that they depend on the PQ charge of the singlet scalar of $SU(2)_L \otimes U(1)_Y$. Using the relations among these PQ charges, we further express the axion state in terms of the original complex scalar components. Our results clarify the structure of PQ symmetry and the scalar sector in the DFSZ model, providing a basis for further studies on axion interaction with matter, the origin of axion properties, and possible phenomenological or experimental signatures.

Keywords: *Peccei-Quinn mechanism, axion-matter interactions, the DFSZ model, the strong CP problem, axion model*

1. Introduction

The discovery of instantons in 1976 led to an explosion of interest in CP violations in strong interactions [1]. This is because, to explain the appearance of instantons in the simplest way, the term $L_\theta = \left(\theta g^2 / 32\pi^2\right) F\tilde{F}$ was added to the classical Lagrangian. However, the term proportional to θ is the cause of CP violation. The problem becomes even more interesting when the experimental limit of the neutron's electric dipole moment predicts that θ can take small values $\theta < 10^{-8}$ [1], [2], [3]. If θ parameter is observable, then this is an accident for strong CP.

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Fortunately, θ is an unobservable parameter and strong interactions automatically conserve CP. Theoretically, the above problem would be solved if we added a symmetry $U(1)$, which would then be broken by from the color anomalies. Following this approach, the best solution was proposed by Peccei–Quinn by introducing Peccei–Quinn models with two scalar doublets of SM [2], [4]. However, Peccei–Quinn models lead to the emergence of axions [5], with evidence suggesting the existence of interactions with matter [6], [7], [8], [9], [10], [11]. This makes the study of axions and their interactions even more fascinating [12], [13], [14].

The DFSZ models become more efficient than the PQ models when a scalar singlet of the $SU(2)_L \otimes U(1)_Y$ group is added. These models naturally explain the origin and very small mass of the axion. Furthermore, they also appear to fit the grand unified theory [15], [16].

In this work, we considered a DFSZ model. We used a special matrix diagonalization method to introduce the mass spectra of scalars, including axions [17], [18], [19], [20]. This also aimed to explain the natural origin of axions in the model. We also studied Peccei–Quinn symmetry to derive the PQ charges of the particles and to determine the state of the axion based on the relationships of the PQ charges.

2. Review of the model

2.1. Particle content

We consider a DFSZ model, extended from the Peccei–Quinn model by adding a scalar singlet of the $SU(2)_L \otimes U(1)_Y$ group. This is a major advantage, as it makes this model both effectively solve the strong CP problem inherited from the Peccei-Quinn model and easy to implement in grand unified theory. Therefore, the fermion part is arranged similarly to the standard model (SM).

$$\Psi_L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} : (1,2,-1), l_R : (1,1,-2), \tag{1}$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} : \left(3,2,\frac{1}{3}\right), u_R : \left(3,1,\frac{4}{3}\right), d_R : \left(3,1,-\frac{2}{3}\right). \tag{2}$$

The scalar part introduces two doublets and one singlet of $SU(2)_L$ group with corresponding hypercharged 1 and 0.

$$\begin{aligned} H_u &= \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}\left(v_u + R_{H_u^0} + iI_{H_u^0}\right) \end{pmatrix} \sim (1,2,1), \\ H_d &= \begin{pmatrix} H_d^+ \\ \frac{1}{\sqrt{2}}\left(v_d + R_{H_d^0} + iI_{H_d^0}\right) \end{pmatrix} \sim (1,2,1), \end{aligned} \tag{3}$$

$$\Phi = \frac{1}{\sqrt{2}}\left(v_\phi + R_\phi + iI_\phi\right) \sim (1,1,0). \tag{4}$$

According to this arrangement, H_u only couples with right-handed quarks with a charge of $2/3$. Conversely, H_d couples with right-handed quarks with charge of $-1/3$ and right-handed leptons. The Yukawa Lagrangian of the model is given:

$$\mathrm{L}^{\mathrm{Yukawa}} = y_u^{H_u}\bar{Q}_L\tilde{H}_u u_R + y_d^{H_d}\bar{Q}_L H_d d_R + y_l^{H_d}\bar{\Psi}_{L_l} H_d l_R + \mathrm{H.c.} \tag{5}$$

where

$$\tilde{H}_u = i\sigma_2 H_u^* = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_u + R_{H_u^0} - iI_{H_u^0}\right) \\ H_u^- \end{pmatrix} \sim (1,2,-1), \tag{6}$$

The Higgs potential of the model is also derived from [8].

$$\begin{aligned} V(\Phi, H_u, H_d) = {} & \lambda_\phi\left(|\Phi|^2 - \mu_\phi^2/2\right)^2 + |H_d|^2\left(\kappa_d|\Phi|^2 - \mu_d^2\right) + |H_u|^2\left(\kappa_u|\Phi|^2 - \mu_u^2\right) \\ & - \left(\kappa\Phi H_u^\dagger H_d + H.c\right) + \lambda_d|H_d|^4 + \lambda_u|H_u|^4 + \lambda\left(|H_u|^2|H_d|^2 - |H_u^\dagger H_d|^2\right). \end{aligned} \tag{7}$$

The Lagrangian of the model is given by

$$\mathcal{L}^{Total} = \frac{1}{2}|\partial_\mu\Phi|^2 + (D_\mu H_d)^\dagger D^\mu H_d + (D_\mu H_u)^\dagger D^\mu H_u + \mathcal{L}^{Yukawa} - V(\Phi, H_u, H_d), \tag{8}$$

where the covariant derivative is defined as

$$D_\mu H_{d,u} \equiv \partial_\mu H_{d,u} - ig\tau.W_\mu H_{d,u} - ig'YB_\mu H_{d,u}, \tag{9}$$

with W_μ and B_μ are the $SU(2)_L$ and $U(1)_Y$ gauge fields, respectively, whose coupling constants are denoted by g and g', and τ correspond to the Pauli matrices.

2.2. Mass spectra of scalars

We can use the minimization of the Higgs potential in Eq. (7), and give relations as follows:

$$\begin{aligned} & 2\lambda_d v^2 c_\beta^2 - \mu_d^2 + \frac{v_\phi^2}{2}\left(\kappa_d - \frac{\kappa t_\beta}{v_\phi}\right) = 0, \\ & 2\lambda_u v^2 s_\beta^2 - \mu_u^2 + \frac{v_\phi^2}{2}\left(\kappa_u - \frac{\kappa}{v_\phi t_\beta}\right) = 0, \\ & \lambda_\phi\left(v_\phi^2 - \mu_\phi^2\right) + 2v^2\left(\kappa_d c_\beta^2 + \kappa_u s_\beta^2 - \frac{\kappa s_{2\beta}}{v_\phi}\right) = 0, \end{aligned} \tag{10}$$

where, we denote $t_\beta = \tan\beta = \frac{v_u}{v_d}, s_\beta = \sin\beta, c_\beta = \cos\beta$.

Assuming that κ_d, κ_u and κ are sufficiently small and do not affect the vacuum stability conditions obtained from other quartic terms of the potential, we have

$$\lambda_\phi > 0, \quad \lambda_d > 0, \quad \lambda_u > 0, \quad \lambda + 2\sqrt{\lambda_d\lambda_u} > 0. \tag{11}$$

First, the charged Higgs boson is given the following mass and state:

$$m_{H^\pm}^2 = 8\left(\lambda v^2 + \frac{\kappa v_\phi}{s_{2\beta}}\right). \tag{12}$$

$$\begin{pmatrix} G_W^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} H_u^\pm \\ H_d^\pm \end{pmatrix}. \tag{13}$$

where $G_W^\pm$ are massless and absorbed by the $W^\pm$ boson.

The pseudo-scalar components, including the axion, are presented as follows:

$$m_A^2 = 8\frac{\kappa}{v_\phi}\left(s_{2\beta}v^2 + \frac{v_\phi^2}{s_{2\beta}}\right), \quad m_a^2 = m_u m_d\left(\frac{f_\pi m_\pi N}{f_a\left(m_u + m_d\right)}\right)^2. \tag{14}$$

$$\begin{pmatrix} a \\ G_Z \\ A \end{pmatrix} = \begin{pmatrix} c_\alpha & 0 & s_\alpha \\ 0 & 1 & 0 \\ -s_\alpha & 0 & c_\alpha \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\beta & s_\beta \\ 0 & -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} I_\Phi \\ I_{H_u^0} \\ I_{H_d^0} \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha s_\beta & s_\alpha c_\beta \\ 0 & c_\beta & s_\beta \\ -s_\alpha & -c_\alpha s_\beta & c_\alpha c_\beta \end{pmatrix}\begin{pmatrix} I_\Phi \\ I_{H_u^0} \\ I_{H_d^0} \end{pmatrix}. \tag{15}$$

where G_Z is massless and absorbed by the Z boson.

$$t_\beta = \frac{v_u}{v_d}, t_\alpha = \frac{v s_{2\beta}}{v_\Phi}, \tag{16}$$

$$a = c_\alpha I_\Phi - s_\alpha s_\beta I_{H_u^0} + s_\alpha c_\beta I_{H_d^0}. \tag{17}$$

$$A = -s_\alpha I_\Phi - c_\alpha s_\beta I_{H_u^0} + c_\alpha c_\beta I_{H_d^0}. \tag{18}$$

The mass matrix of the neutral CP-even scalar fields is given by

$$M = \begin{pmatrix} 32v^2\left(\lambda_d c_\beta^4 + \lambda_u s_\beta^4\right) & M_{12} & M_{13} \\ M_{21} & \frac{8\kappa v_\phi}{s_{2\beta}} + 8v^2\left(\lambda_d + \lambda_u\right)s_{2\beta}^2 & M_{23} \\ M_{31} & M_{32} & 4\lambda_\phi v_\phi^2 \end{pmatrix}, \tag{19}$$

where

$$M_{12} = M_{21} = 16v^2\left(-\lambda_d c_\beta^2 + \lambda_u s_\beta^2\right)s_{2\beta}, \tag{20}$$

$$M_{13} = M_{31} = 8vv_\phi\left(\kappa_d c_\beta^2 + \kappa_u s_\beta^2 - \kappa s_{2\beta} / v_\phi\right), \tag{21}$$

$$M_{23} = M_{32} = -4vv_\phi\left[\left(\kappa_d - \kappa_u\right)s_{2\beta} + 2\kappa c_{2\beta} / v_\phi\right]. \tag{22}$$

Based on Ref.[6,7], we diagonalize the matrix in Eq.(19) using the following rotation:

$$R = \exp\left(\frac{v}{v_\phi}A + \frac{v^2}{v_\phi^2}B\right), \qquad A^T = -A, \qquad B^T = -B. \tag{23}$$

with the non-zero components of matrices A and B given as follows:

$$A_{13} = \frac{2}{\lambda_\Phi}\left(\kappa_d c_\beta^2 + \kappa_u s_\beta^2 - \kappa s_{2\beta}/v_\Phi\right), \qquad A_{23} = \frac{\left(\kappa_d - \kappa_u\right)s_{2\beta} + 2\kappa c_{2\beta}/v_\Phi}{\dfrac{2\kappa}{v_\Phi s_{2\beta}} - \lambda_\Phi}, \tag{24}$$

$$\begin{aligned} B_{12} = &-\frac{2v_\Phi}{\kappa}s_{2\beta}^2\left(\lambda_d c_\beta^2 - \lambda_u s_\beta^2\right) + \frac{v_\Phi s_{2\beta}\left(\kappa - v_\Phi s_{2\beta}\lambda_\Phi\right)}{\lambda_\Phi \kappa\left(2\kappa - v_\Phi s_{2\beta}\lambda_\Phi\right)}\left(\kappa_d c_\beta^2 + \kappa_u s_\beta^2 - \frac{\kappa s_{2\beta}}{v_\Phi}\right) \\ &\times\left(\left(\kappa_d - \kappa_u\right)s_{2\beta} + 2\kappa c_{2\beta}/v_\Phi\right). \end{aligned} \tag{25}$$

Therefore, the masses and states of the CP-even Higgs bosons are given as:

$$\begin{aligned} m_{h_1}^2 &\simeq 32v^2\left(\lambda_d c_\beta^4 + \lambda_u s_\beta^4\right) - \frac{16v^2}{\lambda_\phi}\left(\kappa_d c_\beta^2 + \kappa_u s_\beta^2 - \frac{\kappa}{v_\phi}s_{2\beta}\right)^2, \\ m_{h_2}^2 &\simeq \frac{8\kappa}{s_{2\beta}}v_\phi + 8v^2 s_{2\beta}^2\left(\lambda_d + \lambda_u\right) - 4v^2\frac{\left[\left(\kappa_d - \kappa_u\right)s_{2\beta} + 2\kappa c_{2\beta}/v_\phi\right]^2}{\lambda_\phi - 2\kappa/\left(v_\phi s_{2\beta}\right)}, \\ m_{h_3}^2 &\simeq 4\lambda_\phi v_\phi^2 + 4v^2\frac{\left[\left(\kappa_d - \kappa_u\right)s_{2\beta} + 2\kappa c_{2\beta}/v_\phi\right]^2}{\lambda_\phi - 2\kappa/\left(v_\phi s_{2\beta}\right)} + \frac{16v^2}{\lambda_\phi}\left(\kappa_d c_\beta^2 + \kappa_u s_\beta^2 - \kappa s_{2\beta}/v_\phi\right)^2. \end{aligned} \tag{26}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\beta & s_\beta \\ 0 & -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} R_\Phi \\ R_{H_u^0} \\ R_{H_d^0} \end{pmatrix}. \tag{27}$$

The lighter state h_1 is identified with the SM-like Higgs boson, while h_2 and h_3 are extra heavy neutral CP-even Higgs bosons.

3. PQ charges

Based on the Ref. [9,10], we introduced the rules to determine PQ charges of multiplets and fields of any Beyond Standard Models (BSMs) which include $U(1)_{PQ}$. Note that the PQ charge of a multiplet is an average value of all components' PQ charges in the multiplet. We denote fermion and scalar as u and φ, respectively. Then, behaviors of fermion and scalar fields under $U(1)_{PQ}$ transformation are:

$$u \to u' = e^{i\alpha\gamma_5 PQ_{(u)}}u, \qquad \bar{u} \to \bar{u}' = \bar{u}e^{i\alpha\gamma_5 PQ_{(u)}}, \qquad \varphi \to \varphi' = e^{-i\alpha PQ_{(\varphi)}}\varphi. \tag{28}$$

The right-handed and left-handed components of the transform:

$$u_R \to u'_R = e^{i\alpha\left[PQ_{(u)}\right]} u_R, \quad \bar{u}_R \to \bar{u}'_R = \bar{u}_R e^{-i\alpha\left[PQ_{(u)}\right]},$$
$$u_L \to u'_L = e^{-i\alpha[PQ_u]} u_L, \quad \bar{u}_L \to \bar{u}'_L = \bar{u}_L e^{i\alpha[PQ_u]}. \tag{29}$$

with $PQ_{(x)}$, $(x = \mathrm{u}, \varphi)$, are the PQ charges of fields and u_L, u_R are left-handed and right-handed fermions, respectively. For convenience, the minus sign in the exponent of scalar field φ is used. This helps to briefly describe the transformation of fermions and scalar as

$$PQ_{(u)} = PQ_{(u_R)} = -PQ_{(u_L)} = PQ_{(\bar{u}_L)}, PQ_{\phi*} = -PQ_{\phi} . \tag{30}$$

2.1. PQ charges of fields

With the spectrum of particles introduced in Sec. I, the Yukawa interactions in this model are determined as

$$\mathcal{L}^{\text{Yukawa}} = \mathrm{y_u^{H_u}} \bar{\mathrm{Q}}_\mathrm{L} \mathrm{H_u u_R} + \mathrm{y_d^{H_d}} \bar{\mathrm{Q}}_\mathrm{L} \mathrm{H_d d_R} + \mathrm{y_l^{H_d}} \bar{\Psi}_\mathrm{L} \mathrm{H_d} l_\mathrm{R} + H.c. \tag{31}$$

$$= \mathrm{y_u^{H_u}} \left(\bar{\mathrm{Q}}_\mathrm{L} \mathrm{H_u u_R} + \bar{u}_R H_u^\dagger Q_L \right) + \mathrm{y_d^{H_d}} \left(\bar{\mathrm{Q}}_\mathrm{L} \mathrm{H_d d}_R + \bar{d}_R H_u^\dagger Q_L \right) + \mathrm{y_l^{H_d}} \left(\bar{\Psi}_\mathrm{L} \mathrm{H_d} l_\mathrm{R} + \bar{l}_R H_d^\dagger \Psi_L \right) \tag{32}$$

$$\begin{aligned} &= \mathrm{y_u^{H_u}} \left(\bar{u}_L H_u^0 u_R + \bar{d}_L H_u^- u_R + \bar{u}_R H_u^{0*} u_L + \bar{u}_R H_u^+ d_L \right) \\ &\quad + \mathrm{y}_d^{\mathrm{H}_d} \left(\bar{u}_L H_d^+ d_R + \bar{d}_L H_d^0 d_R + \bar{d}_R H_d^- u_L + \bar{d}_R H_d^{0*} d_L \right) \\ &\quad + \mathrm{y}_l^{\mathrm{H}_d} \left(\bar{v}_L H_d^+ l_R + \bar{l}_L H_d^0 l_R + \bar{l}_R H_d^- \nu_L + \bar{l}_R H_d^{0*} l_L \right). \end{aligned} \tag{33}$$

Based on Eq.(29), we derive the relationship between the PQ charges of the multiplets as follows:

$$\begin{aligned} &PQ_{\bar{Q}_L} + PQ_{H_u} + PQ_{u_R} = 0, \\ &PQ_{\bar{Q}_L} + PQ_{H_d} + PQ_{d_R} = 0, \\ &PQ_{\bar{\Psi}_L} + PQ_{H_d} + PQ_{l_R} = 0. \end{aligned} \tag{34}$$

In the detailed representation, the results applied to neutral scalars are:

$$\begin{aligned} &PQ_{H_u^0} + PQ_{\bar{u}_L} + PQ_{u_R} = 0, \\ &PQ_{H_d^0} + PQ_{\bar{d}_L} + PQ_{d_R} = 0, \\ &PQ_{H_d^0} + PQ_{\bar{l}_L} + PQ_{l_R} = 0. \end{aligned} \tag{35}$$

With charged particles, we have:

$$\begin{aligned} &PQ_{H_u^+} + PQ_{\bar{u}_R} + PQ_{d_L} = 0, \\ &PQ_{H_d^+} + PQ_{\bar{u}_L} + PQ_{d_R} = 0, \\ &PQ_{H_d^+} + PQ_{\bar{v}_L} + PQ_{l_R} = 0. \end{aligned} \tag{36}$$

Replacing Eq. (30) with Eq. (34), we get:

$$PQ_{u_R} = -\frac{1}{2} PQ_{H_u^0}; PQ_{d_R} = -\frac{1}{2} PQ_{H_d^0}; PQ_{l_R} = -\frac{1}{2} PQ_{H_d^0}. \tag{37}$$

Combining Eq.(36) with Eq.(37), we have:

$$PQ_{H_u^+} = -PQ_{H_d^+} = PQ_{u_R} + PQ_{d_R} = PQ_{l_R} - PQ_{\upsilon_L} \quad (38)$$

Furthermore, when we consider the third-order interaction term in the Higgs potential, the relationship is given:

$$PQ_{\Phi} - PQ_{H_u} + PQ_{H_d} = 0. \quad (39)$$

Thus,

$$PQ_{H_u} - PQ_{H_d} = \frac{PQ_{H_u^0} - PQ_{H_d^0}}{2} = PQ_{\Phi}. \quad (40)$$

and

$$PQ_{H_u} = \frac{PQ_{H_u^0} - PQ_{H_u^+}}{2}, PQ_{H_d} = \frac{PQ_{H_d^0} + PQ_{H_d^+}}{2}. \quad (41)$$

From Eq. (17), there is a mixture of $I_{\Phi}, I_{H_u^0}, I_{H_d^0}$ that leads to three orthogonal physical states a, G_Z, A. It is reasonable to assume that the mixing of PQ charges for the two neutral components of the doublet scalars is characterized by a mixing angle β satisfying Eq. (10)

$$PQ_{H_u^0} = 2c_\beta^2 PQ_{\Phi}, PQ_{H_d^0} = -2s_\beta^2 PQ_{\Phi}. \quad (42)$$

We can represent PQ_{H_u} and PQ_{H_d} in terms of PQ_{Φ}.

$$PQ_{H_u} = (c_\beta^2 + \frac{1}{2})PQ_{\Phi}, PQ_{H_d} = (-s_\beta^2 + \frac{1}{2})PQ_{\Phi}. \quad (43)$$

Similarly, we can derive the PQ charges for multiplets of $SU(2)_L$ and fields in Table 1 and Table 2, respectively.

Table 1. The PQ charges of $SU(2)_L$ multiplets.

	$SU(2)_L$ doublets				$SU(2)_L$ singlets			
	Ψ_L	Q_L	H_u	H_d	l_R	u_R	d_R	Φ
PQ	$\frac{1}{2}PQ_\Phi$	$\frac{1}{2}PQ_\Phi$	$\left(c_\beta^2+\frac{1}{2}\right)PQ_\Phi$	$\left(-s_\beta^2+\frac{1}{2}\right)PQ_\Phi$	$s_\beta^2 PQ_\Phi$	$-c_\beta^2 PQ_\Phi$	$s_\beta^2 PQ_\Phi$	PQ_Φ

Table 2. The PQ charges of fields.

	Fermions				**Scalars**			
	l_L	υ_L	u_L	d_L	H_u^0	H_u^+	H_d^0	H_d^+
PQ	$-s_\beta^2 PQ_\Phi$	$\left(c_\beta^2-2s_\beta^2\right)PQ_\Phi$	$c_\beta^2 PQ_\Phi$	$-s_\beta^2 PQ_\Phi$	$2c_\beta^2 PQ_\Phi$	$-\left(c_\beta^2-s_\beta^2\right)PQ_\Phi$	$-2s_\beta^2 PQ_\Phi$	$\left(c_\beta^2-s_\beta^2\right)PQ_\Phi$

2.1. The relationship between pseudo-scalars.

For convenience, we rewrite the scalar field with the condition $I_\Phi \ll R_\Phi, R_\Phi \approx v_\Phi$. Its form is:

$$\Phi_{v_\phi} \equiv \Phi\big|_{R_\phi = v_\phi} = \frac{v_\phi}{\sqrt{2}} e^{i\frac{I_\phi}{v_\phi}}. \tag{44}$$

Under the transformation of $U(1)_Y$, we have:

$$H_u^0 \to H_u^{0\prime} = \frac{v_u}{\sqrt{2}} e^{iY_{H_u^0}\frac{I_{H_u^0}}{v_u}}, \tag{45}$$

$$H_d^0 \to H_d^{0\prime} = \frac{v_d}{\sqrt{2}} e^{iY_{H_d^0}\frac{I_{H_d^0}}{v_d}}. \tag{46}$$

We recall that $Y_{H_u^0} = 1, Y_{H_d^0} = 1$, then from the three-order term of Higgs potential we get:

$$-\frac{I_{H_u^0}}{v_u} + \frac{I_{H_d^0}}{v_d} = 0. \tag{47}$$

$$I_{H_u^0} = \frac{v_u}{v_d} I_{H_d^0} = t_\beta I_{H_d^0}. \tag{48}$$

Moreover, under the transformation of $U(1)_{PQ}$, we get:

$$H_u^{0\prime} \to H_u^{0\prime\prime} = \frac{v_u}{\sqrt{2}} e^{iY_{H_u^0}\frac{I_{H_u^0}}{v_u}PQ_{H_u^0}}, \tag{49}$$

$$H_d^{0\prime} \to H_d^{0\prime\prime} = \frac{v_d}{\sqrt{2}} e^{iY_{H_d^0}\frac{I_{H_d^0}}{v_u}PQ_{H_d^0}}, \tag{50}$$

$$\Phi \to \Phi'' = \frac{v_\phi}{\sqrt{2}} e^{i\frac{I_\phi}{v_\phi}PQ_\phi}. \tag{51}$$

Applying for the three-order term of Higgs potential we get:

$$-Y_{H_u^0}\frac{I_{H_u^0}}{v_u}PQ_{H_u^0}+Y_{H_d^0}\frac{I_{H_d^0}}{v_d}PQ_{H_d^0}+\frac{I_\phi}{v_\phi}PQ_\Phi=0. \quad (52)$$

$$-\frac{I_{H_u^0}}{v_u}PQ_{H_u^0}+\frac{I_{H_d^0}}{v_d}PQ_{H_d^0}+\frac{I_\Phi}{v_\phi}PQ_\Phi=0. \quad (53)$$

Therefore, we can give the following relations:

$$I_{H_u^0}=2\frac{v_u}{v_\Phi}I_\Phi, I_{H_d^0}=2\frac{v_d}{v_\Phi}I_\Phi, \quad (54)$$

Instead of Eq. (17), we obtain the state of the axion which now depends only on the complex component of Φ as follows:

$$a=\left(c_\alpha-s_\alpha s_\beta\frac{v_u}{2v_\Phi}+s_\alpha c_\beta\frac{v_d}{2v_\Phi}\right)I_\Phi. \quad (55)$$

Similarly, we can also represent the state of the axion in terms of the complex component of $I_{H_u^0}$ or $I_{H_d^0}$. This is a consequence of the relationship of the PQ charges between the fields established above.

4. Conclusions

In this work, we investigated the state of the axion in a DFSZ model. We derived the mass spectrum of the Higgs bosons in the model. Notably, we derived the mass and state of CP-even Higgs bosons by using exponential rotation of the matrices and identifying the lightest particle of this type with one belonging to the SM. We demonstrate the relationship between the PQ charges of the fields. Based on this, we derive the state of the axion depending on the original complex scalar components.

The results obtained in this work may suggest further research into the interaction of axions with matter and indicate axion-related signals that can be explored experimentally.

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